Total No. of Questions : 8]

SEAT No. :

P1255

[Total No. of Pages : 4

[5121]-204

M.A./M.Sc. (Semester - II)

MATHEMATICS

MT-604 : Linear Algebra

(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Answer any five questions.*
2) *Figures to the right indicate full marks.*
3) *Use of non-programmable, scientific calculator is allowed.*

Q1) a) Let $(W_i)i \in \Delta$ be a family of subspaces of a vector space V. Then show that, the following are equivalent. **[5]**

i) $\sum\limits_{i\in\Delta} W_i$ is a direct sum.

ii) If $\sum\limits_{i\in\Delta} X_i = 0$ where $x_i \in W_i$ then xi = 0 for all $i \in \Delta$.

iii) $W_i \cap \sum\limits_{\substack{j\in\Delta \\ j\neq i}} W_j = 0$ for all $i \in \Delta$

b) Show that, in a vector space F^n, the set of n-tuples $(x_1, ..., x_i, ..., x_n)$ with $x_i = 0$ is a subspace. **[3]**

c) Complete the set {(2, 1, 4, 3), (2, 1, 2, 0} to form a basis of R^4. **[2]**

Q2) a) Let U, V be vector spaces over F. Let $(e_1, e_2, ... e_n)$ be an ordered basis of U. Given a list $f_1, f_2, ..., f_n$ of elements of V. Prove that, there is a unique linear mapping $f : U \to V$ such that

$f(e_i) = f_i, \quad i = 1, 2, ..., n.$

Further, f is an isomorphism if and only if $(f_1, f_2, ..., f_n)$ is a basis of V. **[5]**

P.T.O.

b) If W is the subspace of V = $\mathbb{R}^4$. generated by e_1 = (1, 2, 3, 4) and e_2 = (0, 0, 0, 1) then find a basis of V/W. **[3]**

c) If $f: R^2 \rightarrow R^2$ is a linear mapping with $f(1,0)=(2,3)$ and $f(0,1)=(-1,1)$ then find $f(a,b)$. **[2]**

Q3) a) Let U, V be vector spaces over F. Then prove that, Hom (U,V) is a vector space over F. Moreover, If dim U = m and dim V = n then dim Hom (U, V) = mn **[5]**

b) Let V be a finite dimensional vector space over F, and let f, g, $\in$ Hom (v, v) such that fg = 1 then show that gf = 1. **[3]**

c) Find the matrix of the linear mapping $\phi: v \rightarrow v$, Where $v = R^{2\times2}$, defined by $\phi(v) = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} v$ with respect to the standard basis $\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$. **[2]**

Q4) a) Let U, V be vector spaces over F with dimensions m, n respectively. Let A be the matrix of a linear mapping $\phi; V \rightarrow U$ with respect to a given pair of ordered bases B, C of U, V respectively. Then prove that, the matrix of ϕ with respect to a new pair of bases B', C' is $A^1 = P^{-1}AQ$.

Where - P, Q are the matrices of transformations from B^1 to B and C^1 to C respectively. **[5]**

b) Show that matrices $A = \begin{pmatrix} -1 & 3 & 0 \\ 0 & 2 & 0 \\ 2 & 1 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ are similar matrices over C. **[3]**

c) Find the matrix associated with the linear map $f: R^4 \rightarrow R^4$ defined by f(a, b, c, d) = (2a, o, b, c+d) with respective to standard bases. **[2]**

Q5) a) Let $\phi \in$ Hom (v, v) and let $f(t)$ be a polynomial over F such that $f(\phi) = 0$. If $f(t) = g(t)h(t)$ is a factorization of $f(t)$ into relatively prime polynomials $g(t)$, $h(t)$ then show that. **[5]**

$$V = \ker g(\Phi) \oplus \ker h(\phi)$$

b) Show that the roots of the characteristic polynomial of the matrix. **[3]**

$$A = \begin{pmatrix} 1 & -2 & -2 & -2 \\ -2 & 1 & -2 & -2 \\ -2 & -2 & 1 & -2 \\ -2 & -2 & -2 & 1 \end{pmatrix}$$

are 3, 3, 3 and –5. Also show that the eigenspaces associated with the eigenvalues 3 and –5 are of dimensions 3 and 1 respectively. **[3]**

c) Determine the eigen values of the matrix. **[2]**

$$A = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}, \text{ if exist.}$$

Q6) a) Reduce the following matrix into triangular form. **[5]**

$$A = \begin{pmatrix} 1 & -2 & 0 \\ 1 & -1 & 2 \\ 0 & 1 & 1 \end{pmatrix}.$$

b) Find the Jordan canonical form of the matrix, **[5]**

$$A = \begin{pmatrix} 5 & 1 & -2 & 4 \\ 0 & 5 & 2 & 2 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 0 & 4 \end{pmatrix}$$

Q7) a) Let V be a vector space over f of characteristic $\neq$ 2 then show that, the mapping $f: B \to Q$ where $Q(x) = B(x, x), x \in V$ from the set of symmetric bilinear forms on V into the set of quadratic forms on V is a 1-1 correspondence. **[5]**

b) Prove that, every finite-dimensional Euclidean vector space has an orthonormal basis. **[5]**

Q8) a) Prove that, quadratic form. **[5]**

$Q(x) = \sum_{i,j=1}^{n} a_{ij} x_i x_j, x = t(x_1, x_2, ..., x_n) \in R^n$ on R^n can be reduced to a diagonal form $Q(x) = \lambda_1 x_1^{1\,2} + \lambda_2 x_1^{1\,2} + ...\lambda n x_n^{1\,2}$ by an orthogonal transformation of co-ordinates where $\lambda_1, \lambda_2 ...., \lambda_n$ are the eigenvalues of A.

b) State and prove sylvester's theorem. **[5]**

● ● ●