Total No. of Questions :8]

SEAT No. :

P1887

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[4921] - 201
M.A. / M.Sc.
MATHEMATICS
MT - 601 : General Topology
(2008 Pattern) (Semester -II)

Time :3 Hours] *[Max. Marks :80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *All questions carry equal marks.*

Q1) a) Define standard topology, lower limit topology and K- topology on $\mathbb{R}$. For each pair of these topologies, explain whether they are comparable, and if so, which is finer. **[8]**

b) Let A be a set. Prove that there is no surjective map $f : \mathrm{A} \to \mathrm{P}(\mathrm{A})$. **[5]**

c) Let X be any set. Let τ be the discrecte topology and τ' be any other topology on X. Determine with justification whether τ is finer than τ' or not. **[3]**

Q2) a) Let X be a topological space. Then prove that **[8]**

i) ϕ and X are closed.

ii) Arbitrary intersections of closed sets are closed.

iii) Finite unions of closed sets are closed.

b) If $\mathcal{B}$ is a basis for the topology of X and $\mathcal{C}$ is a basis for the topology of Y, then prove that the collection $\mathcal{D}\{B\times C \,/\, B\in \mathcal{B} \text{ and } C\in \mathcal{C}\}$ is a basis for the topology of X×Y. **[5]**

c) Define open map. Let X and Y be topological spaces. Show that the projection map $\pi_1 : X\times Y \to X$ is an open map. **[3]**

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Q3) a) Let X and Y be topological spaces; let $f : X \to Y$. Then prove that the following conditions are equivalent: **[8]**

i) f is continuous.

ii) For every subset A of X, $f(\overline{A}) \subseteq \overline{f(A)}$.

iii) For every closed set B of Y, the set f^{-1} (B) is closed in X.

b) Let X be a space satisfying the T_1 axiom; let A be a subset of X. Then prove that the point x is a limit point of A if and only if every neighborhood of x contains infinitely many points of A. **[5]**

c) Show that a subspace of a Hausdorff space is Hausdorff. **[3]**

Q4) a) State and prove the sequence lemma. **[8]**

b) State and prove the pasting lemma. **[5]**

c) Prove that every finite point set in a Hausdorff space is closed. **[3]**

Q5) a) Prove that a finite cartesian product of connected spaces is connected. **[8]**

b) Show that X is Hausdorff if and only if the diagonal $\Delta = \{x \times x : x \in X\}$ is closed in X×X. **[5]**

c) If the sets C and D forms a separation of X, and if Y is a connected subspace of X, then prove that Y lies entirely within either C or D. **[3]**

Q6) a) Prove that a space X is locally connected if and only if for every open set U of X, each component of U is open in X. **[8]**

b) Let $f : X \to Y$ be a bijective continuous function. If X is compact and Y is Hausdorff, then prove that *f* is a homeomorphism. **[4]**

c) Prove that a subspace of a regular space is regular. **[4]**

Q7) a) Prove that compactness implies limit point compactness, but not conversely. **[8]**

b) Prove that every compact Hausdorff space is normal. **[8]**

Q8) a) State and prove the Tychonoff theorem. **[10]**

b) State: **[6]**

i) the Urysohn lemma,

ii) the Tietze extension theorem.

iii) the Urysohn metrization theorem.