Total No. of Questions :8]

SEAT No. :

P1883

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[4921] - 101

M.A. / M.Sc.

MATHEMATICS

MT - 501 : Real Analysis

(2008 Pattern) (Semester - I) (Old)

Time :3 Hours] *[Max. Marks :80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Define a normed linear space and show that C [*a, b*] is a normed linear space with supnorm. **[6]**

b) With usual notations prove that

$\| x \| = \frac{1}{3} \| x \|_1 \ \& \ \frac{2}{3} \| x \|_\infty$ defines a norm on $\mathbb{R}^n$. **[5]**

c) State and prove Arzela Ascori theorem. **[5]**

Q2) a) Let I be an interval in $\mathbb{R}^n$ and $\text{on}(\text{I}) = \underset{k=1}{\overset{n}{\pi}} (b_k - a_k)$ ε be a collection of finite union of disjoint intervals in $\mathbb{R}^n$ then show that m is a measure on ε. **[6]**

b) If $M_{\mathcal{F}}$ denote $\{A_k \subset \mathbb{R}^n / D(A_k, A) \to 0 \text{ as } k \to \infty\}$ for some sequence A_k in ε then prove that $M_{\mathcal{F}}$ is a ring. **[5]**

c) Let A and B be subsets of a metric space (M, *d*). Then prove or dis prove. **[5]**

i) $\text{int}(A \cap B) = \text{int}(A) \cap \text{int}(B)$

ii) $\text{int}(A \cup B) = \text{int}(A) \cup \text{int}(B)$

P.T.O.

Q3) a) Let $f = \mathbb{R}^n \to \mathbb{R} \cup \{\pm \infty\}$ then show that following statements are equivalent. **[6]**

i) $\{x / f(x) > a\}$ is measurable.

ii) $\{x / f(x) \geq a\}$ is measurable for any a $\in \mathbb{R}$.

b) If f is a measurable function then prove that $|f|$ is also measurable. **[5]**

c) If $\{f_k\}$ is a sequence of measurable functions then show that limsup f_k and lim inf f_k are measurable. **[5]**

Q4) a) State and prove monotone convergence theorem. **[6]**

b) If $f \in \mathcal{L}(\mathbb{R}^n)$ then for measurable sets A and B with B $\subseteq$ A and m (A\B) = 0 then show that $\int_A f \text{ dm} = \int_B f \text{ dm}$. **[5]**

c) If f, g $\in \mathcal{L}(\mathbb{R}^n)$ and c $\in \mathbb{R}$, E is a measurable set of $\mathbb{R}^n$, then prove that,

i) $\int_E cf \text{ dm} = c\int_E f \text{ dm}$.

ii) $\int_E (f+g) \text{dm} = \int_E f \text{ dm} + \int_E g \text{ dm}$. **[5]**

Q5) a) Whether a Riemann integrable function is Lebesgue integrable also? Whether converse holds? Justify. **[6]**

b) State and prove Lebesgue dominated convergence theorem. **[5]**

c) State Fatou's lemma and show that strict inequality holds in it. **[5]**

Q6) a) For $1 \leq p < \infty$ prove that $L^p(\mu)$ is a linear space. **[6]**

b) State and prove Holder's inequality. **[5]**

c) Define counting measure and probability measure. **[5]**

Q7) a) Define an orthonormal sequence in $\mathbb{R}^n$ and show that

$\frac{1}{\sqrt{2\pi}}, \frac{\cos nx}{\sqrt{\pi}}, \frac{\sin mx}{\sqrt{\pi}}$ is an orthonormal sequence in $\mathcal{L}^2([-\pi, \pi], m)$. **[8]**

b) Define step functions and show that they are dense in $\mathcal{L}^p(\mu)$ for $1 \leq p < \infty$. **[8]**

Q8) a) State and prove Riesz - Fischer theorem. **[8]**

b) State and prove Bessel's inequality. **[8]**