Total No. of Questions :8]

SEAT No. :

P1411

[Total No. of Pages :3

[5221] - 105
M.A./M.Sc.
MATHEMATICS
MT-505: Ordinary Differential Equations
(2013 Pattern) (Semester - I) (Credit System)

Time : 3 Hours] *[Max. Marks :50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If $y_1(x)$ and $y_2(x)$ are two solutions of the equation $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$, then prove that they are linearly dependent on this interval if and only if their Wronskian $W(y_1, y_2)$ is identically zero. **[5]**

b) Verify that $y_1 = x$ is one solution of differential equation $x^2 y'' + 2xy' - 2y = 0$ and find y_2 and the general solution. **[3]**

c) Show that $y = c_1 e^x + c_2 e^{-x}$ is general solution of $y'' - y = 0$ on any interval. **[2]**

Q2) a) Discuss the method of variation of parameters to find the solution of second order differential equation with constant coefficients. **[5]**

b) Find the general solution of $y'' + 10y' + 25y = 14e^{-5x}$ by using method of undetermined coefficients. **[3]**

c) Change the independent variable x by $x = e^z$ and solve the differential equation $x^2 y'' + 3xy' + 10y = 0$. **[2]**

P.T.O.

Q3) a) State and prove sturm comparison theorem. **[5]**

b) Show that the zeros of the functions asinx + bcosx and csinx + dcosx are distict and occurs alternately if $ad - bc \neq 0$. **[3]**

c) Replace the differential equation $\frac{d^2x}{dt^2} + 4t\frac{dx}{dt} + t^2x = 0$ by an equivalent system of first order equations. **[2]**

Q4) a) Find the general solution of $\left(1+x^2\right)y'' + 2xy' - 2y = 0$ in terms of power series in x. **[5]**

b) Determine the nature of the point $x = \infty$ for the equation $x^2y'' + xy' + \left(x^2 - 4\right)y = 0.$ **[3]**

c) Locate and classify the singular point on the X - axis of $x^2\left(x^2-1\right)^2 y'' - x(1-x)y' + 2y = 0$ **[2]**

Q5) a) Find two independent Frobenius series solutions of the differential equation $4xy'' + 2y' + y = 0$. **[5]**

b) Prove that the function $E(x, y) = ax^2 + bxy + cy^2$ is positive definite if and only if $a > 0$ and $b^2 - 4ac < 0$. **[3]**

c) Find the critical point of the system

$$\frac{dx}{dt} = 2x - 2y + 10$$

$$\frac{dy}{dt} = 11x - 8y + 49$$ **[2]**

Q6) a) Solve the system

$$\frac{dx}{dt} = x + y$$

$$\frac{dy}{dt} = 4x - 2y.$$ **[5]**

b) Prove that $\log(1 + x) = x\, F(1, 1, 2, -x)$. **[3]**

c) State Picard's existence and uniqueness theorem. **[2]**

Q7) a) Find the general solution near x = 0 of the hypergeometric equation $x(1-x)y'' + [c-(a+b+1)x]y' - aby = 0$ where a, b and c are constants. **[5]**

b) If m_1 and m_2 are roots of the auxiliary equation of the system

$$\frac{dx}{dt} = a_1 x + b_1 y$$

$$\frac{dy}{dt} = a_2 x + b_2 y$$

which are real, distinct and of same sign then prove that the critical point (0, 0) is a node. **[5]**

Q8) a) Let F(x, y) be a continuous function that satisfies a Lipschitz condition $|f(x, y_1) - f(x, y_2)| < K|y_1 - y_2|$ on a strip defined by $a \leq x \leq b$ and $-\infty < y < \infty$. If (x_0, y_0) is any point of the strip, then prove that the initial value problem $y' = f(x, y), y(x_0) = y_0$ has one and only one solution on the interval $a \leq x \leq b$. **[5]**

b) Solve the following initial value problem

$$\frac{dy}{dx} = z, \quad y(0) = 1$$

$$\frac{dz}{dx} = -y, \quad z(0) = 0.$$ **[5]**

☼ ☼ ☼