Total No. of Questions :8]

SEAT No. :

P1251

[Total No. of Pages : 3

[5121]-105
M.A./M.Sc (Semester - I)
MATHEMATICS
MT- 505:Ordinary Differential Equations
(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks :50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If $y_1(x)$ and $y_2(x)$ are two solutions of equation $y''+P(x)y'+Q(x)\,y=0$ on [a,b], then prove that their wronskian W (y_1, y_2) is identically equal to zero or never zero on [a,b]. **[5]**

b) Find the general solution of $x^2y''+3xy'+10\,y=0$ **[3]**

c) Show that $y = c_1e^{2x} + c_2xe^{2x}$ is the general solution of $y''-4y'+4y=0$ on any interval. **[2]**

Q2) a) Discuss the method of variation of parameters to find the solution of second order differential equation with constant coefficients. **[5]**

b) Find the general solution of $y'' + 4y = 3\sin x$ by using method of undetermined coefficients. **[3]**

c) Verify that $y_1 = x$ is one solution of differential equation $x^2y''+xy'-y=0$ and find another solution y_2 and the general solution. **[2]**

Q3) a) State and prove Sturm's separation theorem. **[5]**

b) Let $u(x)$ be any non-trivial solution of $u''+q(x)u=0$ where $q(x)>0$ for all $x > 0$. If $\int_1^\infty q(x)\,dx=\infty$, then prove that $u(x)$ has infinitely many zeros on the positive x-axis. **[3]**

c) Find the normal form of Bessel's equation $x^2y''+xy'+(x^2-p^2)\,y=0$. **[2]**

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Q4) a) Find the general solution of the system. **[5]**

$$\frac{dx}{dt} = 7x + 6y$$
$$\frac{dy}{dt} = 2x + 6y$$

b) Find the indicial equation and its root of the differential equation $x^3 y'' + (\cos 2x - 1) y' + 2xy = 0$. **[3]**

c) Locate and classify the singular points on the x–axis of $x^3 (x-1) y'' - 2(x-1) y' + 3xy = 0$. **[2]**

Q5) a) Find the two independent Frobenius series solution of the differential equation $2x^2 y'' + x (2x + 1) y' - y = 0$. **[5]**

b) Prove that the function $E(x,y) = ax^2 + bxy + cy^2$ is positive definite if and only if $a >$ o and $b^2 - 4ac < 0$. **[3]**

c) Find the critical points of **[2]**

$$\frac{dx}{dt} = y^2 - 5x + 6$$
$$\frac{dy}{dt} = x - y$$

Q6) a) Find the general solution near $x = 0$ of the hypergeometric function.

$x (1-x) y'' + [c - (a + b + 1) x] y' - aby = 0$ where a , b and c are constants. **[5]**

b) Prove that $\lim\limits_{a \to \infty} F\left(a, b, b, \frac{x}{a}\right) = e^x$. **[3]**

c) Replace the differential equation $y'' - x^2 y' + xy = 0$ by an equivalent system of first order equation. **[2]**

Q7) a) If m_1 and m_2 are roots of the auxiliary equation of the system.

$$\frac{dx}{dt} = a_1 x + b_1 y$$
$$\frac{dy}{dt} = a_2 x + b_2 y$$

which are real, distict, and of opposite sign, then prove that the critical point (0,0) is a saddle point. **[5]**

b) Find the exact solution of initial value problem $y' = y^2$, $y(0) = 1$, starting with $y_0(x) = 1$. Apply Picard's method to calculate $y_1(x)$, $y_2(x)$, $y_3(x)$ and compare it with exact solution. **[5]**

Q8) a) Show that the function $f(x,y) = xy$ satisfies Lipschitz condition on any rectangle $a \le x \le b$ and $c \le y \le b$; but it does not satisfy a Lipschitz condition on any strip $a \le x \le b$ and $-\infty < y < \infty$. **[5]**

b) Find the general solution of $y'' + (1 + x)\, y' - y = 0$ about $x = 0$ by power series. **[5]**

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