Total No. of Questions : 8]

SEAT No. :

P1906

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[4921]-1005
M.A/M.Sc.
MATHEMATICS
MT - 505 : Ordinary Differential Equations
(2013 Pattern) (Semester-I)(Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions..*
2) *Figures to the right indicate full marks.*

Q1) a) If $y_1(x)$ and $y_2(x)$ are any two solutions of equation $y''+P(x)y'+Q(x)y=0$ on[a,b], then prove that their wronskian $W=W(y_1,y_2)$ is either identically zero or never zero on [a,b]. **[5]**

b) Verify that $y_1=x$ is one solution of $x^2y''+xy'-y=0$ and then find another solution y_2 and general solution. **[3]**

c) Find the solution of the initial value problem

$y''-5y'+6y=0$, $y(1)=e^2$ and $y'(1)=3e^2$ **[2]**

Q2) a) Discuss the method of undetermined coefficients to find the solution of second order differential equation with constant coefficients. **[5]**

b) Find particular solution of differential equation $y''+y=\sec x$ by variation of parameter method. **[3]**

c) Replace the differential equation $y''-x^2y'-xy=0$ by an equivalent system of first order equations. **[2]**

Q3) a) State and prove sturm seperation theorem. **[5]**

b) Find the normal form of Bessel's equation $x^2y''+xy'+(x^2-p^2)y=0$. **[3]**

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c) Locate and classify the singular points on the x-axis of

$$x^2(x^2-1)^2 y''-x(1-x)y'+2y=0$$ **[2]**

Q4) a) Verify that the equation $y''+y'-xy=0$ has a three term recursion formula and find its series solutions $y_1(x)$ and $y_2(x)$ such that $y_1(0)=1, y_1'(0)=0$ and $y_2(0)=0,\ y_2'(0)=1$ **[5]**

b) Find the indicial equation and its roots of the differential equation

$$x^3 y''+(\cos 2x - 1)y' + 2xy = 0.$$ **[3]**

c) Show that e^x and e^{2x} are linearly independent solutions of $y''-3y'+2y=0$ on any interval. **[2]**

Q5) a) Find two independent Frobenius series solution of the differential equation $4xy''+2y'+y=0$. **[5]**

b) Prove that the function $E(x,y)=ax^2+bxy+cy^2$ is of positive definite if and only if $a>0$ and $b^2-4ac<0$. **[3]**

c) Prove that $\lim\limits_{b\to\infty} F\left(a,b,a,\frac{x}{b}\right)=e^x$ **[2]**

Q6) a) Find the general solution of the system.

$$\frac{dx}{dt}=3x-4y \qquad \frac{dy}{dt}=x-y$$ **[5]**

b) Find the nature and stability properties of critical point(0,0) for

$$\frac{dx}{dt}=-4x-y \qquad \frac{dy}{dt}=x-2y.$$ **[3]**

c) Find the critical points of $\frac{d^2x}{dt^2}+\frac{dx}{dt}-(x^3+x^2-2x)=0$. **[2]**

Q7) a) If m_1 and m_2 are roots of the auxiliary equation of the system

$$\frac{dx}{dt}=a_1x+b_1y$$

$$\frac{dy}{dt}=a_2x+b_2y$$

Which are conjugate complex but not pure imaginary, then prove that the critical point (0,0) is a spiral **[5]**

b) Find the general solution of differential equation

$$x(1-x)y''+\left(\frac{3}{2}-2x\right)y'+2y=0$$ near the singular point $x = 0$. **[5]**

Q8) a) Find the exact solution of initial value problem $y'=y^2$, $y(0)=1$, starting with $y_0(x)=1$. Apply Picard's method to calculate $y_1(x)$, $y_2(x)$, $y_3(x)$ and compare it with the exact solution **[5]**

b) Show that $f(x,y)=x^2|y|$ satisfies a Lipschitz condition on the rectangle $|x|\le 1$ and $|y|\le 1$ but that $\frac{\partial f}{\partial y}$ fails to exists at many points of this rectangle. **[5]**

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