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SEAT No. :

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M.A. / M.Sc.

MATHEMATICS

MT- 502: Advanced Calculus

(Semester -I) (2008 Pattern)

Time : 3 Hours] *[Max. Marks :80*

Instructions to the candidates:

1) *Attempt any five questions:*
2) *Figures to the right indicate full marks.*

Q1) a) Let $\bar{f}, \bar{g}\ \ S \to \mathbb{R}^m$ where $S \subset \mathbb{R}^n$ be vector fields and $\bar{a} \in \mathbb{R}^n$.Let $\lim_{\bar{x}\to\bar{a}} \bar{f}(\bar{x}) = \bar{b}$ and $\lim_{\bar{x}\to\bar{a}} \bar{g}(\bar{x}) = \bar{c}$ then prove that $\lim_{\bar{x}\to\bar{a}} \left[\bar{f}(\bar{x}) \cdot \bar{g}(\bar{x})\right] = \bar{b} \cdot \bar{c}$. **[6]**

b) Let $f(x, y) = \dfrac{x\,y^2}{x^2 + y^4}$ if $(x, y) \neq (0,0)$ and $f(0,0) = 0$. Show that all the directional derivatives exist at (0.0) but the function is not continuous at (0,0). **[5]**

c) Evaluate the directional derivative of $f(x, y, z) = \left(\dfrac{x}{y}\right)^z$ at (1, 1, 1) in the direction of $2\bar{i} + \bar{j} - \bar{k}$. **[5]**

Q2) a) Let $f: S \to \mathbb{R}$, $S \subset \mathbb{R}^n$ be a scalar field. Assume that the partial derivatives $D_1 f, \ --- \ D_n f$ exist in some n-ball $B(\bar{a})$ and are continuous at $\bar{a}$, then prove that f is differentiable at $\bar{a}$. **[8]**

b) If a scalar field f is differentiable at $\bar{a}$ then prove that f is continuous at $\bar{a}$. **[4]**

c) Let z be a function of x and y where $x = u^2 + v^2 - 2uv$, $y = u + v$. Compute $(x + y)\dfrac{\partial z}{\partial x} + (x - y)\dfrac{\partial z}{\partial y}$. **[4]**

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Q3) a) Define Line integral and illustrate it by an example. Also state the basic properties of Line integral. **[6]**

b) Let $\overline{f} = (f_1,....,f_n)$ be a continuously differentiable vector field on an open set S in $\mathbb{R}^n$. If $\overline{f}$ is gradient on S, then prove that the partial derivatives of the components of $\overline{f}$ are related by the equation $D_i f_j(\overline{x}) = D_j f_i(\overline{x})$ for $i, j = 1, 2, ..., n$ and every $\overline{x} \in S$. **[5]**

c) Evaluate the line integral of the vector field $\overline{f}(x, y\ z) = (y^2 - z^2)\ \overline{i} + 2yz\ \overline{j} - x^2\overline{k}$, along the path described by $\overline{\alpha}(t) = t\overline{i} + t^2\ \overline{j}\ + t^3\ \overline{k}, 0 \le t \le 1.$ **[5]**

Q4) a) State and prove first fundamental theorem for line integrals . **[8]**

b) Calculate the work done by constant force with help of line integrals .**[4]**

c) Let $\overline{f}$ be a vector field continuous on an open connected set S in $\mathbb{R}^n$? If the Line integral of $\overline{f}$ is zero around every piecewise smooth closed path in S then prove that the line integral of $\overline{f}$ is independent of the path in S. **[4]**

Q5) a) Let $\overline{f}\ (x, y) = P\ (x, y)\ \overline{i}\ + Q\ (x, y)\ \overline{j}$ be a vector field that is continuously differentiable on an open simply connected set S in the plane. Prove that f is a gradient an S if and only if $\frac{\partial p}{\partial y} = \frac{\partial \theta}{\partial x}$ every where on S. **[8]**

b) Transform the given integrals to one or more iterated integrals in polar Co - ordinates $\int_0^1 \left[\int_0^1 f(x, y)\, dy\right] dx$. **[6]**

c) Determine the volume of an n-dimensional interval. **[2]**

Q6) a) State and prove Green's theorem for plane regions bounded by piecewise smooth Jordan curve. **[6]**

b) Make a sketch of the region of integration and evaluate $\iiint_S \sqrt{x^2 + y^2}\, dx\, dy\, dz$ where S is the solid formed by the upper nappe of the cone $z^2 = x^2 + y^2$ and the plane $Z = 1$. **[5]**

c) Evaluate the Line integral using the Green's theorem $\oint_c y^2 dx + x\, dy$ where C is the square with vertices $(\pm 1, \pm 1)$. **[5]**

Q7) a) Define fundamental vector product. Find the fundamental vector product for the surface with explicit representation. What are the singular points of the surface with explicit representation. **[6]**

b) Define the surface integral and explain the terms involved in it. **[5]**

c) Compute the area of the region cut from the plane $x + y + z = a$ by the cylinder $x^2 + y^2 = a^2$. **[5]**

Q8) a) State and prove Gauss divergence theorem **[8]**

b) Determine the Jacobian matrix and compute the curl and divergence of the following vector field $\overline{F}\,(x, y, z) = (x^2 + yz)\,\overline{i} + (y^2 + xz)\,\overline{j} + (z^2 + xy)\,\overline{k}$. **[6]**

c) Define simple parametric surface. **[2]**

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