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SEAT No. :

P 848

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T.Y.B.Sc.

STATISTICS (Principal)

ST - 342 : Testing of Hypotheses

(2013 Pattern) (Paper - II) (Semester - IV) (Theory)

Time : 2 Hour] *[Max. Marks :40*

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*
3) *Use of Calculator and statistical table is allowed.*
4) *Symbols and abbreviations have their usual meaning.*

Q1) Attempt each of the following :

A) Choose the correct alternative in each of the following : **[1each]**

i) For exponential distribution with mean θ, to test null hypothesis $H_o : \theta = \theta_0$ against $H_1 : \theta > \theta_0$, the critical region will be

a) $\sum X_i < C$ b) $\sum X_i > C$

c) $|\bar{X}| < C$ d) $|\bar{X}| > C$

ii) Which of the following is used for testing goodness of fit?

a) Sign test

b) Mann - Whitney U test

c) Kolmogorov- Smirnov test

d) Run Test

iii) For carrying out SPRT which of the following should be fixed in advance?

a) Both α and β b) Only α

c) Only β d) Neither α nor β

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iv) For a random sample of size n from the Bernoulli distribution with parameter θ, the likelihood ratio test for testing the hypothesis $H_0 : \theta = 0.5$ against $H_1 : \theta \neq 0.5$ is to reject H_0 if

a) $\sum X_i^2 < C$ b) $|\bar{X}| < C$

c) $\sum X_i^2 > C$ d) $|\bar{X}| > C$

B) State whether the following statements are true or false : **[1each]**

i) The value of the likelihood ratio statistic close to zero indicates that data supports the null hypothesis.

ii) For testing randomness of the sample run test is used.

C) Define each of the following : **[1each]**

i) Test of hypothesis.

ii) Type I error.

D) Explain each of the following : **[1each]**

i) Power of the test.

ii) Likelihood ratio test.

Q2) Attempt any two of the following : **[5each]**

A) Construct, UMP test of level of significance α for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$ where θ is the mean of exponential distribution based on a random sample of size n drawn from it.

B) Construct SPRT of strength (α, β) for testing, $H_0 : m = m_0$ against $H_1 : m = m_1$ $(m_0 > m_1)$, Where m is the mean of poisson distribution.

C) Let X is a cauchy random variable with location parameter θ and scale parameter $\lambda = 3$. To test the hypothesis $H_0 : \theta = 5$ against $H_1 : \theta = 10$, a single observation is taken. The rejection region is $x > 8$. compute the probabilities of type I error and type II error.

Q3) Attempt any two of the following : **[5 each]**

A) Describe Kolmogorov - Smirnov test for one sample problem

B) Construct a likelihood ratio test of level α for testing the hypothesis $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 < \sigma_0^2$ for a random sample of size n taken from the N $(4, \sigma^2)$ distribution.

C) State Neyman - pearson theorem. Use it to find the most powerful test of level α for testing the hypothesis $H_0 : \theta = 1$ against $H_1 : \theta = 2$ where θ is the parameter of the distribution of a random variable X with p.d.f. given by

$$f(x) = \frac{\theta}{1-\theta} x^{\left(\frac{2\theta-1}{1-\theta}\right)} \quad 0 < x < 1, \theta > 0$$

$= 0$ o.w.

Q4) Attempt any one of the following :

A) i) Construct a SPRT of strength (α, β) for testing the hypothesis $H_0 : \theta = \theta_0$ against the hypothesis $H_1 : \theta = \theta_1 (\theta_1 > \theta_0)$ for Bernoulli distribution with parameter θ. **[5]**

ii) Following is a random sample drawn from the continuous population in the order in which the observations are made :

75, 56, 44, 89, 95, 23, 32, 84, 77, 71, 88, 41.

Test the hypothesis of randomness of the sample. Use 5% level of significance. **[5]**

B) i) Let $X_1, X_2, \text{——} X_n$ denote the random sample of size n from the normal distribution with mean μ and standard deviation 16. Find the sample size n and a uniformly most powerful test of level 0.1 for testing $H_0 : \theta = 25$ against $H_1 : \theta < 25$ with power function $K(\theta)$ such that $k(24) = 0.5$ **[5]**

ii) Steel rods produced by a certain company have a median lenght 10 meters when the process is operating properly. A sample of 10 rods, randomly selected from production line, yields the following observed length.

9.87, 10.18, 10.22, 9.84, 10.05, 9.81, 10.03, 10.09, 9.95, 9.80

Assuming that the lengths are symmetrically distributed about their median, test whether the process is operating properly using Wilcoxon's signed ranked test. (Use 5% . l.o.s.) **[5]**

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