Total No. of Questions : 4]

SEAT No. :

P801

[Total No. of Pages : 3

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T.Y. B.Sc. (Semester - IV)

MATHEMATICS

MT - 341 : Complex Analysis

(2013 Pattern) (Semester - IV) (Paper - I)

Time : 2 Hours] ***[Max. Marks : 40***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any five of the following : **[10]**

a) Find the principal argument Arg z when $z = \left(\sqrt{3} - i\right)^6$.

b) Show that limit of the function $f(z) = \left(\frac{z}{\bar{z}}\right)^2$ as z tends to 0 does not exist.

c) Find singular points for the function $f(z) = \frac{z^2 + 1}{(z+2)(z^2 + 2z + 2)}$.

d) Show that when $n = 0, \pm 1, \pm 2, \ldots\ldots\ldots, (-1)^{\frac{1}{\pi}} = e^{(2n+1)i}$

e) Evaluate $\int_C f(z)dz$ where $f(z) = \frac{z+2}{z}$ and C is the semicircle $z = 2e^{i\theta} (0 \le \theta \le \pi)$.

f) Find $\int_C f(z)dz$ where $f(z) = \frac{z}{e^z}$ and C is the circle |z| = 1 in the positive sense.

g) Show that the singular point of the function $f(z) = \frac{1 - \cosh z}{z^3}$ is a pole. Determine the order m of the pole and corresponding residue B.

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Q2) Attempt <u>any two</u> of the following : **[10]**

a) Suppose that $f(z) = u(x, y) + iv(x, y)$ and that $f'(z)$ exists at a point $z_0 = x_0 + iy_0$. Then, show that the first order partial derivatives of u and v must exist at (x_0, y_0) and they must satisfy the Cauchy Riemann equations $u_x = v_y, u_y = -v_x$ there. Also, show that $f'(z_0) = u_x + iv_x$ where the partial derivatives are calculated at (x_0, y_0).

b) Let $f(z)$ be an analytic function in a domain D such that $|f(z)|$ is constant throughout D. Prove that $f(z)$ must be constant throughout D.

c) Find the four roots of the equation $z^4 + 4 = 0$ and use them to factor $z^4 + 4$ into quadratic factors with real coefficients.

Q3) Attempt <u>any two</u> of the following : **[10]**

a) Show that $u(x, y) = \sinh x \sin y$ is harmonic in some domain and find it's harmonic conjugate $v(x, y)$.

b) Show that for all z a) $\overline{\sin z} = \sin \overline{z}$ b) $\overline{\cos z} = \cos \overline{z}$.

c) Write the two Laurent series in powers of z that represent the function $f(z) = \dfrac{1}{z(1+z^2)}$ in certain domain and specify those domains.

Q4) Attempt <u>any one</u> of the following : **[10]**

a) i) State and prove the Cauchy integral formula.

ii) Show that if C is the boundary of the triangle with vertices at the points 0, 3i and –4 oriented in the counterclockwise direction then

$$\left|\int_C (e^z - \overline{z})dz\right| \leq 60.$$

b) i) Let f be continuous in a domain D. If $\int_C f(z)dz = 0$ for every closed contour lying in D, then show that f is analytic throughout D.

ii) Using the Cauchy's residue theorem evaluate the integral of $f(z) = \frac{z+1}{z^2 - 2z}$ around the circle $|z| = 3$ in the positive sense.

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