Total No. of Questions : 4]

SEAT No. :

P901

[Total No. of Pages : 3

[5315]-504
T.Y.B.Sc. (Principal)
STATISTICS (Theory)
Distribution Theory - I
(2008 Pattern) (Semester - III) (Paper - I)

Time : 2 Hours] ***[Max. Marks : 40***

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*
3) *Use of scientific calculator and statistical tables is allowed.*
4) *Symbols and abbreviations have their usual meaning.*

Q1) Attempt each of the following :

A) Choose the correct alternative in each of the following **[1 each]**

a) β_1(m,n) distribution is symetric about $\left(\frac{1}{2}\right)$ if

i) m > n ii) m < n

iii) m = n iv) m = n + 1

b) If $(X_1, X_2, X_3) \sim MD\,(12, 0.1, 0.4, 0.5)$, then karl peason's coefficient of correlation between X_2 and X_3 is

i) $-\sqrt{\frac{2}{3}}$ ii) $-\sqrt{\frac{3}{2}}$

iv) $\sqrt{\frac{2}{3}}$ vi) –0.02

c) Mean of $W(\alpha = 3, \beta = 1)$ distribution is

i) 1 ii) 3

iii) 2 iv) $\frac{1}{3}$

d) The cumulative distribution function of 1st order statistic $X_{(1)}$ of a random sample of site n from the distribution of a r.v.x is

i) $[F(x)]^n$ ii) $1-[F(x)]^n$

iii) $1-n[F(x)]^{n-1}$ iv) $1-[1-F(x)]^n$

P.T.O.

B) State whether each of the following statement is true or false : **[1 each]**

a) If $X \sim \beta_1\,(5,6)$, then mean of $\left[\frac{1-X}{X}\right]$ is 1. **[1]**

b) If $X \sim W\,(4,2)$, then $\frac{X^2}{16}$ follows exponential distribution. **[1]**

c) Define convergence in distribution **[1]**

d) State chebychev's inequality. **[1]**

e) State the additive property of cauchy distribution **[1]**

f) Define order statistics **[1]**

Q2) Attempt any two of the following: **[5 each]**

a) Let $X \sim W\,(\alpha,\beta)$, obtain the cumulative distribution function of X and hence find first and third quartile of X.

b) Let $X \sim \beta_2\,(m,n)$, obtain mean and variance of X.

c) State and prove central limit theorem for i.i.d. random variables.

Q3) Attempt any two of the following: **[5 each]**

a) Obtain the probability distribution of i^{th} order statistic of a random sample of size n drawn from a distribution.

b) Let $(x_1,x_2,x_3) \sim MD\,(12, 0.4, 0.2, 0.4)$.

Compute

i) $P(x_1 = 2, x_3 = 6)$

ii) $\text{Corr}\,(2x_1 + 3, x_2 + 2)$

c) If X is a random variable with $E(x) = 5$ and $E(x^2) = 34$. Obtain the upper limit for $p[-1 < x < 11]$ using chebychev's inequality.

Q4) Attempt any one of the following:

a) i) Find the expectation of sample median drawn from U(0,1) distribution when sample size $n = (2m + 1)$ where m is non-negative integer.

ii) If $\{X_k\}$ is a sequence of independent r.v. each assuming three values $-1, 0, 1$ with the respective probabilities $P[X_k = -1] = P[X_k = 1] = \frac{1}{k}$ and $P[X_k = 0] = 1 - \frac{2}{k}$. Verify WLLN for this. **[6+4]**

b) i) State and prove the inter relation between $\beta_1(m,n)$ and $\beta_2(m,n)$ distributions.

ii) Eight independent observations are taken on a r.v. X having the p.d.f. $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$

Suppose the interval (0,1) is divided into 4 equal parts.
Find the probability that equal number of observations lie in each of these parts. **[6+4]**

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