Total No. of Questions : 4]

P712

SEAT No. :

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[5315] - 301

T.Y.B.Sc.

MATHEMATICS

MT - 331 : Metric Spaces

(2013 Pattern) (Semester - III) (Paper - I) (91113)

Time : 2 Hours]   *[Max. Marks : 40*

Instructions to the candidates:

    1)   *All questions are compulsory.*

    2)   *Figures to the right indicate full marks.*

Q1) Attempt any five of the following: **[10]**

    a)   Does $d(x,y) = |\sin(x+y)|$ define a metric on $\mathbb{R}$? Justify.

    b)   In a discrete metric space $\mathbb{R}_d$, find $S_3(2)$ and $S_{\frac{1}{3}}[2]$.

    c)   State true or false with justification: If A and B are subsets of $\mathbb{R}_u$ such that $\overline{A} \subset \overline{B}$ then $A \subset B$.

    d)   Let $A = \left\{ \frac{1}{n} \middle/ n \in \mathbb{N} \right\}$ be a subset of $\mathbb{R}_u$. Find ∂A (boundary of A).

    e)   Let $Y = (0,1)$ be a subspace of $\mathbb{R}_u$ and $A = \left(0, \frac{1}{2} \right]$. State with justification whether A is closed in Y.

    f)   Give an example of compact set which is not connected and connected set which is not compact.

    g)   Is the set $A = \left\{ \frac{1}{n} \middle/ n \in \mathbb{N} \right\} \cup \{0\}$ nowhere dense in $\mathbb{R}_u$? Justify.

Q2) Attempt any two of the following: **[10]**

    a)   In a metric space (X,*d*), prove that the arbitrary intersection of closed sets in X is closed.

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b) Let A and B be subsets of a metric space (X,d). Show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$. Is it true that $\overline{A \cap B} = \overline{A} \cap \overline{B}$? Justify.

c) Let (X,d) be metric space. Show that $|d(x,z)-d(z,y)| \leq d(x,y)$ for all $x,y,z \in X$.

Q3) Attempt any two of the following: **[10]**

a) Let $f:(X,d) \rightarrow (Y,\rho)$ be a homomorphism. Prove that $F \subset X$ is closed in X if and only if $f(F)$ is closed in Y.

b) Let (Y,d_Y) be a subspace of complete metric space (X,d). If Y is closed then prove that Y is complete.

c) Let $\mathbb{C}_u$ be usual metric space with metric $d(z_1,z_2)= |z_1-z_2|; z_1,z_2 \in \mathbb{C}$ (set of complex numbers). Show that $\mathbb{C}_u$ is complete.

Q4) Attempt any one of the following:

a) i) Prove that a metric space (X,d) is compact if and only if every collection of closed subsets of X having FIP (finite intersection property) has non-empty intersection.   **[7]**

    ii) Prove that any finite subset of metric space (X,d) is compact. **[3]**

b) i) Let (X,d) be metric space and $Y \subset X$ is connected. If $Z \subset X$ is such that $Y \subset Z \subset \overline{Y}$ then prove that Z is connected. Hence prove that $\overline{Y}$ is connected.   **[5]**

    ii) Let (X,d) be complete metric space and $A \subset X$. Prove that $\overline{A}$ is compact if and only if A is totally bounded.   **[5]**

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