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SEAT No. :

P715

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T.Y.B.Sc.

MATHEMATICS

MT-334: Group Theory

(2013 Pattern) (Semester - III) (Paper - IV) (91143)

Time : 2 Hours] *[Max. Marks : 40*

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any Five of the following: **[10]**

a) Let m and n be positive integers and consider a subgroup $H=\{mx+ny \mid x, y\in \mathbb{Z}\}$ of a group $\mathbb{Z}$. Show that H is a cyclic subgroup of $\mathbb{Z}$.

b) Find the maximum possible order for an element of S_{15}.

c) Let $\mu = (1, 2, 4, 5)\ (3, 6)$ in S_6. Find the index of a subgroup $<\mu>$ in S_6.

d) Let G be a finite group with identity e and let $a\in G$. Show that there exists a positive integer n such that $a^n = e$.

e) Determine the condition under which the map $\phi : G \to G$ defined by $\phi(x) = x^{-1}, \forall x\in G$ is a group homomorphism?

f) Show that a group of prime order is a simple group. Also, give an example of a non-cyclic simple group.

g) Give an example of a group G having no element of finit order > 1, but having a factor group G/H, all of whose elements are of finite order.

P.T.O.

Q2) Attempt any Two of the following: **[10]**

a) Show that a non-empty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H$, for all $a, b \in H$.

b) Let $\phi : G \to G'$ be a group homomorphism with kernel H. Prove that $\phi[G]$ is a group and is isomorphic to G/H.

c) Show that there is no permutation σ such that $\sigma^{-1}(1,2,3)\,\sigma = (1,3)\,(5,7,8)$.

Q3) Attempt any Two of the following: **[10]**

a) Prove that a subgroup of a cyclic group is cyclic.

b) Prove that every permutation σ of a finite set is a product of disjoint cycles.

c) Show that the set of $n \times n$ matrices with determinant one form a normal subgroup of GL(n, $\mathbb{R}$).

Q4) Attempt any one of the following:

a) i) Is the converse of the Lagrange theorem true in general? Justify. **[4]**

ii) Let $\phi : G \to G'$ be a group homomorphism. Prove that ϕ is a one-to-one map if and only if Ker(ϕ) = $\{e\}$.
Also, show that Ker(ϕ) is a normal subgroup of G. **[6]**

b) i) Show that the binary structure $\langle \mathbb{R}, + \rangle$ is isomorphic to the structure $\langle \mathbb{R}^{+}, \cdot \rangle$. **[4]**

ii) Prove that the group $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic if and only if m and n are relatively prime. **[6]**

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