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T.Y.B.Sc.

MATHEMATICS

MT-337(B): Dynamical Systems

(2013 Pattern) (Semester - III) (Paper - VII) (911B3)

Time : 2 Hours]

[Max. Marks : 40

Instructions to the candidates:

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*

Q1) Atmpt any Five of the following:

[10]

- a) Is the equilibrium point $(0, 0)$ a saddle point for the system $X' = \begin{bmatrix} -2 & 1 \\ -1 & 1 \end{bmatrix} X$?

Justify.

- b) Find the eigenvalues and eigenvectors of $\exp(A)$ if $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

- c) Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 1 & 3 \\ \sqrt{2} & 3\sqrt{2} \end{bmatrix}$.

- d) Give an example of a system of differential equations for which $(t, 1)$ is a solution for $t > 0$.

- e) For which values of k and b is the system $X' = \begin{bmatrix} 0 & 1 \\ -k & -b \end{bmatrix}$ a center?

Justify.

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f) Find the stable and unstable line of the system $X' = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} X$.

g) If $\lambda, \mu \in \mathbb{R}$ then show that, $\exp \begin{bmatrix} \lambda & 0 \\ 0 & \mu \end{bmatrix} = \begin{bmatrix} e^\lambda & 0 \\ 0 & e^\mu \end{bmatrix}$.

Q2) Attempt any two of the following: **[10]**

a) If V_0 is an eigenvector of $A_{n \times n}$ with associated eigenvalue λ , then show that $X(t) = e^{\lambda t} V_0$ is a solution of the system $X' = AX$.

b) Show that $x^2 - y^2 = 1$ is a solution of the system $X'(t) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} X(t)$ passing through $(1, 0)$.

c) Find the general solution of $X' = AX$ and sketch the phase portrait if

$$A = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix}.$$

Q3) Attempt any two of the following: **[10]**

a) Let A be an $n \times n$ matrix. Show that the initial value problem $X' = AX$ with $X(0) = X_0 \in \mathbb{R}^n$ has unique solution $X(t) = \exp(tA)X_0$.

b) Let A be a 3×3 matrix for which λ is the only eigenvalue. If $\ker(A - \lambda I) = 2$ then show that there exists a 3×3 matrix T such that,

$$T^{-1}AT = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}.$$

c) Find the general solution of $X' = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix} X$ and sketch the phase portrait of the system.

Q4) Attempt any two of the following:

[10]

- a) Let A be a 2×2 matrix which has two real distinct eigenvalues λ_1, λ_2 with associated eigenvectors V_1, V_2 . Show that $T^{-1}AT = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$ is the canonical form of matrix A .

- b) Find the canonical form of $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 1 \\ -1 & -1 & 2 \end{bmatrix}$.

- c) Find the exponential of $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

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