Total No. of Questions :5]

SEAT No. :

P597

[Total No. of Pages : 4

[5315]-14
F.Y.B.Sc.
STATISTICS/STATISTICAL TECHNIQUES
Discrete Probability and Probability Distributions
(2013 Pattern) (Paper - II)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*
3) *Use of statistical tables and calculator is allowed.*
4) *Symbols have their usual meanings.*

Q1) Attempt each of the following:

a) i) Define a discrete sample space. **[1]**

ii) Give one real life situation where binomial distribution can be applied. **[1]**

iii) State mode of a Poisson distribution with parameter m=3. **[1]**

iv) If A and B are independent events with P(A)=0.4, P(B)=0.5, find $P(A' \cap B)$ **[1]**

b) Choose the correct alternative for each of the following: **[1 each]**

i) Relative complement of A with respect to B is given by:

A) $A \cap B'$ B) $A' \cup B$

C) $A' \cap B$ D) $A' \cap B'$

ii) In the simulteneous tossing of two fair coins, the probability of having at least one head is:

A) 0.75 B) 0.25

C) 1 D) 0.5

iii) If X is a degenerate random variable then:

A) $E(X^2) < [E(X)]^2$ B) $E(X^2) > [E(X)]^2$

C) $E(X^2) = [E(X)]^2$ D) None of these

iv) Let X_1 and X_2 are two independent Poisson variates then the probability distribution of (X_1+X_2) is :

A) Binomial B) Poisson

C) Geometric D) Bernoulli

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c) i) State the multiplication theorem for two events A and B defined on a sample space Ω. **[2]**

ii) The probability distribution of a discrete random variable X is as follows:

x	0	2	4
$P(X=x)$	0.25	0.40	0.35

Find E(x) **[2]**

iii) Define binomial distribution. **[2]**

iv) Determine K such that the following function is a probability mass function (p.m.f)

$$P(X=x) = kx \; ; \; x=1, 2, 3, 4.$$
$$=0 \quad ; \text{otherwise}$$ **[2]**

Q2) Attempt any four of the following: **[4 each]**

a) An integer between 1 and 100 (both inclusive) is selected at random. Find the probability of selecting a perfect square. Also find the probability of selecting a perfect cube, if all integers are equally likely.

b) Let X $\rightarrow$ B $(n=6, p=\frac{1}{4})$

Find i) $P(X=3)$

ii) $P(X<3)$

c) X and Y are random variables with joint probability mass function partly shown in the following table. Find the missing probabilities.

X \ Y	0	1	Total
0	$\frac{1}{6}$	-	-
1	-	-	$\frac{2}{3}$
Total	-	$\frac{1}{2}$	1

d) Let X be a descrete random variable with probability mass function,

$$P(X=x)=\frac{x}{15}, \quad \text{for } x=1, 2, 3, 4, 5.$$
$$= 0, \quad \text{otherwise}$$

Find E(X) and Var (2X-3)

e) Given A and B are two independent events defined on Ω, prove that A' and B' are independent.

f) Explain the following terms with one illustration each:
   i) mutually exclusive events
   ii) exhaustive events.

Q3) Attempt any Four of the following: **[4 each]**

a) Obtain moment generating function (m.g.f) of Poisson distribution with parameter 'm'.

b) Give the classical definition of probability. State its limitations.

c) A bag contains 4 white and 2 black balls. Another bag contains 3 white and 3 black balls. One ball is drawn from each bag at random. Find the probability that they are of different colours.

d) The probability mass function (p.m.f) of a random variable X is given by,

$$P(X=x)=\frac{k2^x}{x!}, \qquad x=0,1,2,3,\ldots\ldots, k>0$$
$$= 0, \qquad \text{otherwise}$$

Find the value of K and $P(X \geq 1)$.

e) Let X~B(n,p). Find mean of X.

f) For a bivariate discrete r.v.(X,Y):

$\sigma_X^2 = 9,\ \sigma_Y^2 = 4,$ Cov(X,Y)=4.

Find i) Var (2X-3Y)
   ii) Cov (2X, 3Y)

Q4) Attempt any Two of the following:

a) i) State and prove Baye's theorem **[6]**
   ii) Define r^{th} order factorial moment of a discrete r.v. **[2]**

b) The joint probability distribution of X and Y is,

X \ Y	1	2	3
0	0.1	0.05	0.15
1	0.05	0.1	0.15
2	0.15	0.15	0.1

Find i) E (Y/X=1) **[4]**
   ii) V (Y/X=1) **[4]**

c) State and prove binomial approximation to hypergeometric distribution. **[8]**

d) The probability distribution of r.v.X is given by, **[8]**

X	0	1	2	3
$P(x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$

Calculate coefficient of skewness γ_1 and comment on the nature of the distribution.

Q5) Attempt any One of the following:

a) i) A r.v. X has discrete uniform distribution with p.m.f, **[6]**

$$P(x)=\frac{1}{n+1} \quad ; \quad x=0,1,.......n.$$
$$=0 \quad ; \quad \text{otherwise}$$

Find mean and variance of X.

ii) Define Bernoulli distribution with parameter p. **[2]**

iii) The joint p.m.f of (X,Y) is as given below: **[8]**

X \ Y	0	1	2
1	0.1	0.2	0.2
2	0.1	0.3	0.1

Find correlation coefficient between X and Y

b) i) Let X be a discrete r.v. with p.m.f

$$P(x)= pq^x \quad , x=0, 1, 2......, 0 < p <1, q=1-p$$
$$= 0 \quad , \text{otherwise}$$

Find E(X) and Var (X), also show that E (X) < Var (X) **[8]**

ii) Given that $P(A_1)=P(A_2)=P(A_3)=\frac{1}{3}$ and $P(B/A_1)=\frac{2}{7}$ $P(B/A_2)=\frac{4}{9}$, $P(B/A_3)=\frac{1}{5}$, find $P(A_2/B)$. **[4]**

iii) A box of 20 mangoes contain 4 rotten mangoes. Two mangoes are drawn at random without replacement from this box. Obtain the probability distribution of the number of rotten mangoes in the sample. **[4]**

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