

Total No. of Questions : 5]

SEAT No. :

**P511**

**[4917]-114**

**[Total No. of Pages : 4**

**F. Y. B. Sc.**

**STATISTICS / STATISTICAL TECHNIQUES**  
**Discrete Probability and Probability Distributions**  
**(2013 Pattern) (Paper - II)**

**Time : 3 Hours]**

**[Max. Marks : 80**

**Instructions to the candidates:**

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.
- 3) Use of Statistical tables and calculator is allowed.
- 4) Symbols have their usual meanings.

**Q1) A)** Attempt each of the following:

- a) Give one real life situation where geometric distribution can be applied. **[1]**
- b) Write the sample space of a random experiment of tossing 3 coins. **[1]**
- c) State moment generating function (m.g.f.) of a binomial distribution with parameters  $n$  and  $p$ . **[1]**
- d) If A and B are independent events with  $P(A)=0.6$  and  $P(B)=0.5$ , find  $P(A \cap B)$ . **[1]**

**B)** Choose correct alternative for the following:

**[1 each]**

- a) If  $P(A \cap B)=0$ , then the two events A and B are
  - i) Exhaustive events
  - ii) Dependent events
  - iii) Mutually exclusive events
  - iv) Independent events
- b) If X and Y are independent random variables with m.g.f.  $M_X(t)$  and  $M_Y(t)$  respectively then,  $M_{X+Y}(t)$  is :
  - i)  $M_X(t) + M_Y(t)$
  - ii)  $M_X(t) - M_Y(t)$
  - iii)  $M_X(t) / M_Y(t)$
  - iv)  $M_X(t) * M_Y(t)$

**P.T.O.**

- c) If  $X \sim P(m)$  then variance of  $X$  is
- i)  $2m$
  - ii)  $m$
  - iii)  $4m$
  - iv)  $0$
- d) If  $X$  and  $Y$  are two random variables with means  $E(X)=2$  and  $E(Y)=3$ , then  $E(2X-3Y)$  is :
- i)  $-5$
  - ii)  $-1$
  - iii)  $13$
  - iv)  $5$

C) a) If Karl Pearson's coefficient of correlation between  $X$  and  $Y$  ( $\rho(x,y)$ ) is 0.6, then find  $\rho\left(\frac{x+2}{-5}, \frac{y-3}{2}\right)$ . [2]

b) Let  $X \sim B\left(n=10, p=\frac{1}{3}\right)$  then state the mode of  $X$ . [2]

c) For a probability distribution of  $X$ , obtain cumulative distribution function (c.d.f.) of  $X$ . [2]

$x$	-1	0	1
$p(x)$	1/2	1/4	1/4

d) State Baye's theorem. [2]

**Q2)** Attempt any four of the following: [4 each]

- a) Explain the following terms with one illustration each.
  - i) Mutually exclusive events.
  - ii) Sure event.
- b) Let  $X$  and  $Y$  are two independent discrete r.v's. show that  $E(XY)=E(X) \cdot E(Y)$
- c) If  $A$  and  $B$  are mutually exclusive and non empty events defined on sample space  $\Omega$ , show that  $P(A|A \cup B) = \frac{P(A)}{P(A)+P(B)}$ .
- d) For a Poisson distribution with  $P(X=2)=P(X=3)$ , compute  $P(X \geq 1)$ .

e) If  $P(A)=0.4$ ,  $P(B)=0.6$  and  $P(A \cup B)=0.8$ , find

i)  $P(A \cap B)$

ii)  $P(A' \cap B)$

f) Define the following terms:

i) Pairwise independence of three events.

ii) Partition of the sample space.

**Q3)** Attempt any four of the following:

**[4 each]**

a) Give the classical definition of probability. State its limitations.

b) Define a discrete uniform distribution with parameter ' $n$ '. Find its mean and variance.

c) Let  $X \sim B(n,p)$ . State the c.g.f. of  $X$ , hence find mean of  $X$ .

d) The probability mass function (p.m.f.) of a r.v.  $X$  is given by,

$$P(X=x) = K {}^5C_x \quad ; x=0,1,2,3,4,5.$$

$$, K > 0$$

$$= 0 \quad ; \text{otherwise.}$$

Find the value of  $K$ .

e) A committee of 4 persons has to be formed out of 5 graduate and 4 non graduate persons. Find the probability that the committee consists of at most 2 non graduate persons.

f) State and prove multiplication theorem of probability for two events  $A$  and  $B$  defined on sample space  $\Omega$ .

**Q4)** Attempt any two of the following:

a) The joint probability distribution of  $(X,Y)$  is given below:

X \ Y	0	1	2	3
-1	0.05	0.05	0.075	0.075
0	0.10	0.25	0.075	0.075
1	0.10	0.05	0.05	0.05

- Find
- i)  $E(Y|X = 0)$  and [4]
  - ii)  $V(Y|X = 0)$ . [4]
- b) i) Obtain variance of a linear combination of two variables X and Y;  
 $V(aX + bY)$ . [6]
- ii) What do you mean by a deterministic model? State one example of it. [2]
- c) State and prove binomial approximation to hypergeometric distribution. [8]
- d) For a certain probability distribution if mean = 5, Variance = 2, coefficient of skewness = +1 and coefficient of kurtosis = +1, find first four raw moments of the distribution. [8]

**Q5)** Attempt any one of the following:

- a) i) The joint p.m.f. of (X,Y) is given by,
- $$P(x,y) = c(x^2+y^2) \quad ; c>0, x = -1,1.$$
- $$y = -2,2.$$
- $$= 0 \quad ; \text{otherwise.}$$
- Obtain
- I)  $c$
  - II) Marginal p.m.f.'s of X and Y
  - III) Are X and Y independent? Justify. [8]
- ii) Let X and Y be two independent Poisson random variables with mean 3 and 2 respectively. Find: [8]
- I)  $P(X=4|X+Y=5)$
  - II)  $E(X|X+Y=5)$
- b) i) If the probability that a certain test yields a positive reaction is equal to 0.4. What is the probability that less than 5 negative reactions occur before the first positive one? [5]
- ii) State the p.m.f. of hypergeometric distribution. Find mean and variance of the distribution. [8]
- iii) What is the probability that a non-leap year should have fifty three Sundays? [3]

