Total No. of Questions : 5]

SEAT No. :

P584

[Total No. of Pages : 3

[5315] - 1
F.Y.B.Sc.
MATHEMATICS
MT - 101 : Algebra and Geometry
(2013 Pattern) (Paper - I)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any eight of the following: **[16]**

a) State principle of strong induction.

b) Define partition of a non-empty set. Write down any one partition of a set $S=\{2,3,4,8\}$.

c) Use remainder theorem to compute the remainder when $f(x)=x^4-3x^3-7x^2-2$ is divided by $g(x)=x-3$.

d) Find eigen vector of the matrix $\begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix}$ corresponding to eigen value 4.

e) Reduce the matrix $A=\begin{bmatrix} 2 & 3 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{bmatrix}$ to echelon form. Hence find rank of A.

f) Discuss the nature of the conic $5x^2-6xy+5y^2+18x-14y+9=0$.

g) Show that the lines $\frac{x+3}{2}=\frac{y+5}{3}=\frac{z-7}{-3}$ and $\frac{x+1}{4}=\frac{y+1}{4}=\frac{z+1}{-1}$ are coplanar.

h) Find equation of the plane passing through the point (2,3,5) and perpendicular to the line whose d.r.s. are 3, –2, 6.

i) Find centre and radius of the sphere $x^2+y^2+z^2+2x+4y+6z+5=0$.

j) Define cone and generator of cone.

P.T.O.

Q2) Attempt any four of the following: **[16]**

a) Using principle of Mathematical induction prove that $1\cdot2+2\cdot3+3\cdot4+........+n(n+1)=\frac{n(n+1)(n+2)}{3}$ for every positive integer n.

b) Let a,x,y be integers and $m>0$ be an integer. If $ax\equiv ay(\text{mod } m)$ and $(a,m)=1$ then prove that $x\equiv y(\text{mod } m)$.

c) Show that $x-\alpha$ is a factor of $f(x)$ in R[x] if and only if $f(\alpha)=0$.

d) Solve the following system by Gauss Jordan method.

$3x + y + 2z = 3$

$2x - 3y - z = -3$

$x + 2y + z = 4$

e) Find g.c.d. of 306 and 657. Also find integers x,y such that $(306,657)=306x+657y$.

f) Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}3 & -1\\ 1 & 1\end{bmatrix}$. Hence find A^{-1}.

Q3) Attempt any two of the following: **[16]**

a) Prove that for any two non-zero integers a,b have unique positive g.c.d., $d=(a,b)$ and can be expressed in the form $d=ma+nb$ for some integers m and n.

b) i) Solve the equation $x^3-9x^2+23x-15=0$; whose roots are in A.P.

ii) Let a,b be integers and $m>0$ be an integer. If $(a,m)=1$ and $m|ab$ then prove that $m|b$.

c) i) Find the values of λ for which the following system admits infinite solutions

$2x + y = 3$

$x - z = \lambda$

$y + 2z = 1$

ii) Show that the square of any integer is of the form 4K or 8K+1.

Q4) Attempt any four of the following: **[16]**

a) Shift the origin to the point (–1,2) and transform the equation $x^2+y^2+2x+4y=0$.

b) Find the angle between the line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$, where l,m,n are d.c.s. of a line and the plane $ax+by+cz+d=0$.

c) Obtain the equation of a plane in normal form.

d) Find the co-ordinates of points where the line $\frac{x+3}{4}=\frac{y+4}{3}=\frac{z-8}{-5}$ intersect the sphere $x^2+y^2+z^2+2x-10y-23=0$.

e) Find equation of the tangent plane at $P(x_1,y_1,z_1)$ to the sphere $x^2+y^2+z^2=a^2$.

f) Find equation of a cone with vertex at $(-1,1,2)$ and guiding curve $3x^2-y^2=1,\ z=0$.

Q5) Attempt any two of the following: **[16]**

a) Without shifting the origin, if due to rotation of axes, the expression $ax^2+2hxy+by^2$ is transformed to $a'x'^2+2h'x'y'+b'y'^2$ then prove that $a+b=a'+b'$ and $ab-h^2=a'b'-h'^2$.

b) i) Find the co-ordinates of the centre and radius of circle $x^2+y^2+z^2-2x-4y+2z-30=0,\ 2x-y+2z-7=0$.

ii) Find the shortest distance between the lines $\frac{x-3}{1}=\frac{y-4}{1}=\frac{z+1}{1}$ and $\frac{x+6}{2}=\frac{y+5}{4}=\frac{z-1}{-1}$.

c) i) Find equation of the plane containing the line $\frac{x+2}{2}=\frac{y+3}{3}=\frac{z-4}{-2}$ and the point (0,6,0).

ii) Find the equation of a right circular cylinder where axis is the line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$, where l,m,n are d.r.s. of the line and r is the radius of cylinder.

ζ ζ ζ

http://www.sppuonline.com