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F.Y. B.Sc. (Computer Science) EXAMINATION, 2017

MATHEMATICS

Paper II

(MTC-102 : Algebra and Calculus)

(2013 PATTERN)

Time : Three Hours **Maximum Marks : 80**

N.B. **:—** (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Use of non-programmable scientific calculator is allowed.

1. Attempt any *eight* of the following : [16]

(*i*) Let

$$R = \{(1, 1), (1, 2), (2, 3), (1, 3), (3, 1), (3, 2)\}$$

be a relation on a set $A = \{1, 2, 3\}$. Draw a digraph of R.

(*ii*) If a, b, c, d are integers such that $a | b$ and $c | d$, then prove that $ac | bd$.

(*iii*) Let Q be the set of rational numbers. A binary operation $*$ on Q is defined as $a * b = ab + 1$. Determine whether $*$ is associative.

P.T.O.

(*iv*) Define a group.

(*v*) Discuss the continuity of function $f(x)$ defined by :

$$f(x) = \frac{x - |x|}{x}$$

at $x = 0$.

(*vi*) Let $f : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = 2x - 1$ and $g : \mathbf{R} \to \mathbf{R}$ be defined by $g(x) = x^2$, find $g \circ f(x)$.

(*vii*) If $y = a^{mx}$, then find nth derivative of y.

(*viii*) State Taylor's theorem with Lagrange's form of remainder.

(*ix*) What are the possible values of rank for a matrix $A_{3\times 4}$?

(*x*) State Cauchy mean value theorem.

2. Attempt any *four* of the following : [16]

(*i*) Let R be a relation on **Z** defined by xRy if and only if $5x + 6y$ is divisible by 11, for $x, y \in \mathbf{Z}$.

Show that R is an equivalence relation on **Z**.

(*ii*) Let a function $f : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = 2x - 3$. Show that f is bijective. Also find inverse of f.

(*iii*) Express the following permutation σ in S_7 as a product of disjoint cycles and hence find order of σ :

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 4 & 1 & 5 & 2 & 7 & 6 \end{pmatrix}$$

Also determine whether σ is even or odd.

(*iv*) Show that $\sqrt{5}$ is not a rational number.

(*v*) Prove that for any two integers a and b, $a \equiv b \pmod{n}$ if and only if a and b leave the same remainder when divided by n.

(*vi*) Find the remainder of 9^{394} when divided by 11.

3. Attempt any *two* of the following : [16]

(*i*) Obtain the transitive closure of R defined on a set A = $\{a, b, c, d\}$ by using Warshall's algorithm where :

$$R = \{(a, a), (a, d), (b, b), (c, d), (d, b), (d, d)\}.$$

(*ii*) Write the composition table for $(z_8, +_8)$ and

(*a*) Find the order of all elements.

(*b*) Find all subgroups.

(*c*) Is it a cyclic group ? If yes, write all generators.

(*iii*) Find greatest common divisor d of 4999 and 1109. Hence find integers m and n such that :

$$d = 4999m + 1109n.$$

4. Attempt any *four* of the following : [16]

(*i*) Verify the Lagrange's mean value theorem for the function :

$$f(x) = \log(x)$$

on $[1, e]$.

(*ii*) Find the values of α and β if the function $f(x)$ is continuous in $(-2, 3)$ where :

$$f(x) = \begin{cases} 4x+5, & \text{if} \quad -2 < x < 0 \\ 2x+\alpha, & \text{if} \quad 0 \le x < 1 \\ x-3\beta, & \text{if} \quad 1 \le x < 3 \end{cases}.$$

(*iii*) Find *n*th derivative of :

$$y = \frac{1}{x^2 - x - 2}.$$

(*iv*) Expand $\log(\sin(x))$ in ascending power of $(x-3)$.

(*v*) Solve the linear system :

$$x - y - z + 2w = 1$$

$$2x + y + 4z + w = 1$$

$$3x + y + 5z + 4w = -3 .$$

(*vi*) Reduce the following matrix A to row echelon form :

$$A = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 4 & 6 & 8 \\ 3 & 5 & 7 & 9 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Find rank of A.

5. Attempt any *two* of the following : [16]

(*i*) State and prove Rolle's theorem. Verify Rolle's theorem for function :

$$f(x) = x^2 - 2x + 1$$

in [0, 2].

(*ii*) Solve by LU decomposition method :

$$x + 2y - 4z = -4$$

$$2x + 5y - 9z = -10$$

$$3x - 2y + 3z = 11.$$

(*iii*) (*a*) If $y = \tan^{-1}(x)$, then prove that :

$$(1 + x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0.$$

(*b*) Evaluate :

$$\lim_{x \to 0}\left(\frac{a}{x} - \cot\left(\frac{x}{a}\right)\right).$$