Total No. of Questions—**5**] [Total No. of Printed Pages—**4**

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F.Y. B.Sc. (Computer Science) EXAMINATION, 2017

MATHEMATICS

Paper II

(MTC-102 : Algebra and Calculus)

(2013 PATTERN)

Time : Three Hours **Maximum Marks : 80**

***N.B.* :—** (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Use of single memory, non-programmable, scientific calculator is allowed.

1. Attempt any *eight* of the following : [16]

(1) Give an example of a relation on a set A = {1, 2, 3} which is reflexive, symmetric but not transitive.

(2) If a, b and c are integers such that $a|b$, $b|c$, then prove that $a|c$.

(3) Define monoid. Give an example.

(4) Examine the continuity of the function $f(x)$ at $x = 0$ where :

$$f(x) = \frac{|x|}{x}.$$

P.T.O.

(5) If $y = (ax + b)^m$, then find the nth derivative of y.

(6) State Maclaurin's theorem with Lagrange's form of remainder.

(7) Which elements of $(Z_6, \bullet)$ satisfy $x^2 = x$?

(8) State first principle of mathematical induction.

(9) Draw the diagraph of the relation :

R = {(1, 2), (3, 4), (4, 2), (1, 4)} on the set A = {1, 2, 3, 4}.

(10) Determine the values of 'a' for which the following system has infinitely many solutions :

$$(a - 3)x + y = 0$$
$$x + (a - 3)y = 0.$$

2. Attempt any *four* of the following : [16]

(1) If R is the relation of set A = {1, 2, 3, 4} defined as $x\mathrm{R}y$ if and only if $x \leq y$, then draw diagraph of relation R and write the matrix of R.

(2) Let $a, b, c, d \in \mathbf{Z}$. If $a \equiv b(\mathrm{mod}\, n)$ and $c \equiv d(\mathrm{mod}\, n)$, then prove that :

(*i*) $(a + c) \equiv (b + d)(\mathrm{mod}\, n)$

(*ii*) $ac \equiv bd(\mathrm{mod}\, n)$.

(3) Find the remainder of 7^{483} when divided by 13.

(4) If p is prime and a, b are integers such that $p \mid ab$, then prove that $p \mid a$ or $p \mid b$.

(5) Let **Z** be the set of all integers. Given $a, b \in Z$, define $a \sim b$ if $a - b$ is an even integer. Then prove that $\sim$ is an equivalence relation.

(6) Determine whether the given permutation is even or odd. Also find the order of σ and σ^{-1}.

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$$

3. Attempt any *two* of the following : [16]

(1) Using Warshall's algorithm obtain transitive closure of relation :

R = {(1, 2), (2, 2), (2, 4), (3, 2), (3, 4), (4, 1)} on the set A = {1, 2, 3, 4}.

(2) Let G = **Z**, the set of integers. Define the binary operation * as, $a * b = a + b - 2$, $a, b \in \mathbf{Z}$.

Show that $\langle \mathbf{Z}, * \rangle$ is an abelian group.

(3) Show that 4999 and 1109 are relatively prime. Also find m & n such that $4999m + 1109n = 1$.

4. Attempt any *four* of the following : [16]

(1) Verify Rolle's theorem for the function :

$$f(x) = \frac{\sin x}{e^x} \text{ on } [0, \pi].$$

(2) Expand $\sin x$ in ascending power of $(x - \pi/2)$.

(3) Solve the following system of linear equations by Gauss elimination method :

$$3x + y + 2z = 3$$
$$2x - 3y - z = -3$$
$$x + 2y + z = 4.$$

(4) Discuss the continuity of $f(x)$ at $x = 0$ where :

$$f(x) = \frac{e^{1/x} - 1}{e^{1/x} + 1}, \qquad x \neq 0$$

$$= 0, \qquad x = 0$$

(5) Find the nth derivative of $y = \dfrac{2x + 3}{x^2 + 3x + 2}$.

(6) Find column rank of the following matrix :

$$A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}.$$

5. Attempt any *two* of the following : [16]

(1) State Leibnitz's theorem and prove that if $y = (\sin^{-1} x)^2$, then :

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0.$$

(2) Solve by LU decomposition method :

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8.$$

(3) (*a*) State and prove Cauchy's mean value theorem.

(*b*) If in the Cauchy's mean value theorem $f(x) = e^x$, $g(x) = e^{-x}$ show that c is arithmetic mean between a & b.