



[4161] – 101

Seat No.	
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F.E. Semester – I Examination, 2012
ENGINEERING MATHEMATICS – I
(2008 Pattern)

Time : 3 Hours

Max. Marks : 100

- Instructions :**
- I) Attempt Q. 1 or Q. 2, Q. 3 or Q. 4, Q. 5 or Q. 6 from Section I and Q. 7 or Q. 8, Q. 9 or Q. 10, Q. 11 or Q. 12 from Section II.
 - II) Answers to the **two** Sections should be written in **separate** books.
 - III) Neat diagrams must be drawn wherever necessary.
 - IV) Black figures to the right indicate full marks.
 - V) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.
 - VI) Assume suitable data, if necessary.

SECTION – I

1. A) Define Rank of the matrix. Find the rank of matrix A by reducing it to its normal form. 6

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$$

- B) Show that the system 5

$$3x + 4y + 5z = \alpha$$

$$4x + 5y + 6z = \beta$$

$$5x + 6y + 7z = \gamma$$

is consistent only when α, β, γ are in geometric progression.

P.T.O.



C) Verify Cayley Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix} \text{ and hence find } A^{-1}. \quad 7$$

OR

2. A) Find Eigen values and Eigen vectors for the matrix 7

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

B) Examine whether the following vectors are linearly dependent. If so find the relation between them $x_1 = (1, 1, 1, 3)$, $x_2 = (1, 2, 3, 4)$, $x_3 = (2, 3, 4, 7)$. 6

C) Given the transformation

$$Y = \begin{bmatrix} 4 & -5 & 1 \\ 3 & 1 & -2 \\ 1 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Find the co-ordinates (x_1, x_2, x_3) corresponding to $(2, 9, 5)$ in Y. 5

3. A) Prove that

$$\frac{1 + \cos \alpha + i \sin \alpha}{1 - \cos \alpha + i \sin \alpha} = \left(\cot \frac{\alpha}{2} \right) \left[e^{i \left(\alpha - \frac{\pi}{2} \right)} \right]. \quad 5$$

B) Solve the equation $x^7 - x^4 + x^3 - 1 = 0$ by using De Moivre's theorem. 5



C) Prove that

$$\log\left[\frac{1}{1-e^{i\theta}}\right] = \log\left(\frac{1}{2}\operatorname{cosec}\frac{\theta}{2}\right) + i\left(\frac{\pi}{2} - \frac{\theta}{2}\right). \quad 6$$

OR

4. A) If z_1, z_2 and origin represent vertices of an equilateral triangle on the Argand diagram, show that 6

$$\frac{1}{z_1^2} + \frac{1}{z_2^2} = \frac{1}{z_1 z_2}$$

B) If $\operatorname{cosec}\left(\frac{\pi}{4} + ix\right) = u + iv$ then prove that

$$(u^2 + v^2)^2 = 2(u^2 - v^2). \quad 5$$

C) If $a = e^{i\alpha}, b = e^{i\beta}, c = e^{i\gamma}$ then prove that

$$\frac{(a+b)(b+c)(c+a)}{abc} = 8 \cos\left(\frac{\alpha-\beta}{2}\right) \cos\left(\frac{\beta-\gamma}{2}\right) \cos\left(\frac{\gamma-\alpha}{2}\right). \quad 5$$

5. A) If $y = \frac{1}{(x-2)(x-1)^2}$

then find n^{th} order differential coefficient of y w.r.t. x . 5

B) If $x = \tan(\log y)$ prove that,

$$(1+x^2)y_{n+2} + [2(n+1)x-1]y_{n+1} + [n^2-n-2]y_n = 0. \quad 5$$



C) Test convergence of the series (**any one**) : 6

1) $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{9}} + \frac{1}{\sqrt{28}} + \frac{1}{\sqrt{65}} + \dots$

2) $\frac{1}{2}x + \frac{1}{2} \cdot \frac{3}{4}x^2 + \frac{1.3.5}{2.4.6}x^3 + \dots$

OR

6. A) Find n^{th} order differential coefficient of y w.r.t. x 5

$$y = \tan^{-1} \left[\frac{\sqrt{1+x^2} - 1}{x} \right]$$

B) If $y = (\sin^{-1}x)^2$ then prove that

$$(1 - x^2) y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0. \quad 5$$

C) Test convergence of the series (**any one**) : 6

1) $\frac{1}{2} - \frac{2}{5} + \frac{3}{10} - \frac{4}{17} + \dots$

2) $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$

SECTION – II

7. A) Using Taylor's theorem express

$$(x - 2)^4 - 3(x - 2)^3 + 4(x - 2)^2 + 5 \text{ in powers of } x. \quad 5$$

B) Expand $(1 + \sin x)^{1/2}$ upto x^6 . 5



C) Solve **any one** :

6

1) Evaluate $\lim_{x \rightarrow 0} \left(\frac{\sinh x}{x} \right)^{1/x^2}$

2) Evaluate $\lim_{x \rightarrow \frac{1}{2}} \frac{\cos^2 \pi x}{e^{2x} - 2xe}$

OR

8. A) Show that $\frac{x}{e^x - 1} = 1 - \frac{x}{2} + \frac{x^2}{12} - \frac{x^4}{720} + \dots$

5

B) Expand $\tan^{-1} x$ in powers of $(x - 1)$ as far as the term in $(x - 1)^3$.

5

C) Solve **any one** :

6

1) Evaluate $\lim_{x \rightarrow 0} \frac{\sin x \cdot \sin^{-1} x - x^2}{x^6}$

2) Find a and b if $\lim_{x \rightarrow 0} \frac{a \cos x - a + bx^2}{x^4} = \frac{1}{12}$

9. Attempt **any two** :

16

A) If $f(x, y) = \frac{1}{x^2} + \frac{1}{2xy} + \frac{\log x - \log y}{x^2 + y^2}$ then prove that

$$xf_x + yf_y + 2f = 0.$$

B) If $x = r \cosh \theta$, $y = r \sinh \theta$ then show that

$$(x - y) (z_x - z_y) = rz_r - z_\theta$$



C) If $u = \log(x^3 + y^3 - x^2y - xy^2)$ prove that

$$u_{xx} + 2u_{xy} + u_{yy} = \frac{-4}{(x+y)^2}$$

OR

10. Solve **any two** :

16

A) If $x = u \tan v$, $y = u \sec v$ prove that

$$(u_x)_y (v_x)_y = (u_y)_x (v_y)_x.$$

B) If $u = f(x^2 - y^2, y^2 - z^2, z^2 - x^2)$ show that

$$\frac{1}{x}u_x + \frac{1}{y}u_y + \frac{1}{z}u_z = 0.$$

C) If $x^m + y^m = b^m$ then prove that

$$\frac{d^2y}{dx^2} = -(m-1)b^m \frac{x^{m-2}}{y^{2m-1}}$$

11. A) If $u = x + y + z$, $uv = y + z$, $uvw = z$, show that

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$$

6

B) Discuss the maxima and minima of $f(x, y) = x^3 + y^3 - 3axy$.

6

C) Examine for functional dependence $u = \frac{x+y}{1-xy}$, $v = \tan^{-1}x + \tan^{-1}y$. Find the relation between them if functionally dependent.

6

OR



12. A) For the transformation $x = u(1 - v)$, $y = uv$ prove that

$$\frac{\partial(x,y)}{\partial(u,v)} \cdot \frac{\partial(u,v)}{\partial(x,y)} = 1. \quad 6$$

B) Divide 120 into three parts so that the sum of their products taken two at a time shall be maximum. 6

C) Find the percentage error in the area of an ellipse when the errors of 2% and 3% are made in measuring its major and minor axis respectively. 6
