

Total No. of Questions—12]

[Total No. of Printed Pages—8+1

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S.E. (CIVIL) (I Semester) EXAMINATION, 2010

ENGINEERING MATHEMATICS-III

(2008 COURSE)

Time : Three Hours

Maximum Marks : 100

N.B. :— (i) Answer 3 questions from Section I and 3 questions from Section II.

(ii) Answers to the two Sections should be written in separate answer-books.

(iii) Neat diagrams must be drawn wherever necessary.

(iv) Figures to the right indicate full marks.

(v) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(vi) Assume suitable data, if necessary.

SECTION I

1. (a) Solve any three : [12]

(i) $(D^2 + 3D + 2)y = e^{e^x} + \operatorname{cose}^x$

(ii) $(D^2 - 4D + 4)y = e^x \cos^2 x$

P.T.O.

$$(iii) \frac{d^2y}{dx^2} + 4y = \tan 2x \text{ [By variation of parameters]}$$

$$(iv) x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10 \left(x + \frac{1}{x} \right)$$

(b) Solve : [5]

$$\frac{dx}{dt} + y = e^t$$

$$x - \frac{dy}{dt} = e^{-t},$$

given that $x = 1, y = 0$ at $t = 0$.

Or

2. (a) Solve any *three* : [12]

$$(i) (D^2 + 6D + 9)y = \frac{1}{x^3 e^{3x}}$$

$$(ii) (D^5 - D)y = 2x + 2^x$$

$$(iii) (D^2 - 4D + 4)y = e^{2x} \sec^2 x$$

[By variation of parameters]

$$(iv) (x + a)^2 \frac{d^2y}{dx^2} - 4(x + a) \frac{dy}{dx} + 6y = x$$

(b) Solve : [5]

$$\frac{dx}{x(y^2 - z^2)} = \frac{dy}{-y(z^2 + x^2)} = \frac{dz}{z(x^2 + y^2)}$$

3. (a) The differential equation satisfied by a beam, uniformly loaded with one end fixed and second subjected to a tensile force P is given by :

$$EI \frac{d^2y}{dx^2} - Py = \frac{-Wx^2}{2}$$

Show that the elastic curve for the beam under conditions

$$y = 0 \text{ and } \frac{dy}{dx} = 0.$$

when $x = 0$, is given by :

$$y = \frac{W}{2P} \left[x^2 + \frac{2}{n^2} - \frac{e^{nx}}{n^2} - \frac{e^{-nx}}{n^2} \right]$$

where, $\frac{P}{EI} = n^2$. [8]

- (b) A homogeneous rod of conducting material of length 100 cm with ends kept at zero temperature satisfies the equation :

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

If the initial temperature is :

$$u(x, 0) = \begin{cases} x & ; 0 \leq x \leq 50 \\ 100 - x & ; 50 \leq x \leq 100. \end{cases} \quad [8]$$

Or

4. (a) It is found experimentally that a weight of 3 kg stretches a spring to 15 cm. If the weight is pulled down 10 cm below equilibrium position and then released

(i) find the amplitude, period and frequency of motion.

(ii) determine the position, velocity and acceleration as a function of time. [8]

(b) Solve the equation : $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

subject to the following conditions :

(i) $u(x, \infty) = 0$

(ii) $u(0, y) = 0$

(iii) $u(1, y) = 0$

(iv) $u(x, 0) = x(1 - x)$ for $0 < x < 1$. [8]

5. (a) Solve the following system of equations by Gauss-Seidel iteration method : [9]

$$9x_1 + 2x_2 + 4x_3 = 20$$

$$x_1 + 10x_2 + 4x_3 = 6$$

$$2x_1 - 4x_2 + 10x_3 = -15$$

(b) Use Runge-Kutta method of fourth order to solve :

$$\frac{dy}{dx} = \sqrt{x + y} ; y(0) = 1$$

to find y at $x = 0.2$ taking $h = 0.1$. [8]

Or

6. (a) Solve the equation :

$$\frac{dy}{dx} = x^2 + y ; y(0) = 1$$

to find y at $x = 0.1$ using Euler's modified method taking $h = 0.05$. [9]

(b) Solve the following system of equations by Cholesky's method :

$$\begin{aligned} 4x_1 - 2x_2 &= 0 \\ -2x_1 + 4x_2 - x_3 &= 1 \\ -x_2 + 4x_3 &= 0. \end{aligned} \quad [8]$$

SECTION II

7. (a) Compute the first four moments, coefficient of skewness and kurtosis for the following frequencies : [6]

No. of Jobs completed	No. of Workers
0—10	6
10—20	26
20—30	47
30—40	15
40—50	6

- (b) Compute the coefficient of correlation between the supply and price : [6]

x	y	f
5	7	6
9	9	9
15	14	13
19	21	20
24	23	16
28	29	11
32	30	7

- (c) There are 6 married couples in a room. If two persons are chosen at random, find the probability that :
- (i) they are of different sex
- (ii) they are married to each other. [5]

Or

8. (a) Obtain the correlation between population density (per square mile) and death rate (per thousand persons) from the data related to 5 cities. [6]

Population Density	Death Rate
200	12
500	18
400	16
700	21
300	10

(b) If two lines of regressions are $9x + y - \lambda = 0$ and $4x + y = \mu$ and the means of x and y are 2 and -3 respectively, find the values of λ and μ and coefficient of correlation between x and y . [6]

(c) Number of road accidents follows a Poisson's distribution with mean 5, find the probability that in a certain month number of accidents on the highway will be : [5]

(i) less than 3

(ii) between 3 and 5

(iii) more than 3.

9. (a) A particle describes the straight line $r = a \sec \theta$ with constant angular velocity ω . Find the radial and transverse components of velocity and acceleration. [5]

(b) If the directional derivatives of $\phi = a(x + y) + b(y + z) + c(x + z)$ has maximum value 12 in the direction parallel to the line

$$\frac{x - 1}{1} = \frac{y - 2}{2} = \frac{z - 1}{3}$$

find the values of a, b, c . [5]

(c) Establish any two : [6]

(i) If $\rho \bar{\mathbf{E}} = \nabla \phi$, prove that $\bar{\mathbf{E}} \cdot \text{curl } \bar{\mathbf{E}} = 0$.

(ii) Show that $\text{curl curl curl curl } \bar{\mathbf{E}} = \nabla^4 \bar{\mathbf{E}}$, where $\bar{\mathbf{E}}$ is solenoidal.

(iii) $\nabla \cdot (r^3 \bar{\mathbf{r}}) = 6r^3$.

Or

10. (a) A particle moves along the curve $x = a \cos t$, $y = a \sin t$; $z = bt$ with constant angular velocity ω . Find the radial and transverse components of its linear velocity and acceleration at any time t . [5]

(b) Find the directional derivatives of ∇f at $(1, 2, -1)$ where $f(x, y, z) = x^2y + xyz + z^3$ along normal to the surface $x^2y^3 = 4xy + y^2z$ at the point $(1, 2, 0)$. [5]

(c) Establish any two :

(i) $\nabla^4 e^r = e^r + \frac{4}{r} e^r$

(ii) $\bar{\mathbf{F}} = \frac{\bar{\mathbf{a}} \times \bar{\mathbf{r}}}{r^n}$ is solenoidal field

(iii) $\nabla \times [\bar{\mathbf{a}} \times (\bar{\mathbf{b}} \times \bar{\mathbf{r}})] = \bar{\mathbf{a}} \times \bar{\mathbf{b}}$

where $\bar{\mathbf{a}}$ and $\bar{\mathbf{b}}$ are constant vectors. [6]

11. (a) Verify Green's theorem for $\vec{F} = xi + y^2j$ over first quadrant of the circle $x^2 + y^2 = 1$. [6]

(b) Evaluate $\iint_S (x\hat{i} + y\hat{j} + z^2\hat{k}) \cdot d\vec{S}$

where S is the curved surface of the cylinder $x^2 + y^2 = 4$ bounded by planes $z = 0$ and $z = 2$. [6]

(c) Evaluate using Stokes' theorem $\int_C (ydx + zdy + xdz)$, where C is intersection of $x^2 + y^2 + z^2 = a^2$, $x + z = a$. [5]

Or

12. (a) Evaluate $\iint_S (2xy\bar{i} + yz^2\bar{j} + xz\bar{k}) d\vec{S}$ over the surface of the region bounded by $x = 0$, $y = 0$, $y = 3$, and $x + 2z = 6$. [6]

(b) Obtain the equation of streamlines in case of steady motion of fluid defined by $\vec{q} = (y - xz)\bar{i} + (yz + x)\bar{j} + (x^2 + y^2)\bar{k}$. [6]

(c) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ for $\vec{F} = (2y + 3)\bar{i} + xz\bar{j} + (4z - x)\bar{k}$ along the path $x^2 = 2t^2$; $y = t$; $z = t^3$ from $t = 0$ to $t = 1$. [5]