Total No. of Questions—**12**] [Total No. of Printed Pages—**8+3**

Seat No.	

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S.E. (Civil) (First Semester) EXAMINATION, 2014

ENGINEERING MATHEMATICS—III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) Answer from Section I Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**.

From Section II Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12**.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of non-programmable electronic pocket calculator is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve the following (any *three*) : [12]

(*i*) $(D^2 + 3D + 2)\, y = e^{e^x}$

P.T.O.

(*ii*) $(D^2 + 6D + 9)\ y = 5^x - \log 2$

(*iii*) $(D^2 + 1)\ y = \cot x$ (By variation of parameters)

(*iv*) $x^2 \dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 3y = x^2 \sin(\log x)$.

(*b*) Solve : [5]

$$\frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{xe^{x^2+y^2}}.$$

Or

2. (*a*) Solve the following (any *three*) : [12]

(*i*) $(D^2 - 4D + 4)\ y = e^{2x} \sin 3x$

(*ii*) $(D^2 + 6D + 9)\ y = e^{-3x}\ (x^3 + \sin 3x)$

(*iii*) $(D^2 - 1)\ y = x \cos 3x$

(*iv*) $(2x + 3)^2 \dfrac{d^2y}{dx^2} + (2x + 3)\dfrac{dy}{dx} - 2y = 24x^2$

(*b*) Solve the simultaneous equations : [5]

$$\frac{dx}{dt} - wy = a \cos pt, \text{ and}$$

$$\frac{dy}{dt} + wx = a \sin pt.$$

3. (*a*) Solve : [8]

$$\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2},$$

if

(*i*) u is finite; $\forall t$

(*ii*) $u(0, t) = 0; \forall t$

(*iii*) $u(L, t) = 0; \forall t$

(*iv*) $u(x, 0) = u_0; 0 \leq x \leq t$

where, L being a length of a bar.

(*b*) The whirling speed of a shaft of length l is given by :

$$\frac{d^4 y}{dx^4} - m^4 y = 0,$$

where

$$m^4 = \frac{Ww^2}{EIg}$$

and y is the displacement at distance x from one end. If both the ends of the shaft are short bearing, show that shaft will whirl when $\sin ml = 0$. [8]

Or

4. (*a*) Weight of 1 N stretches a spring 5 cm, a weight of 3 N is attached to the spring and weight W is pulled to 10 cm below the equilibrium position and released. Determine the position and velocity. [8]

(*b*) A string is stretched and fastened to two points L aparts. Motion is started by displacing the string in the form :

$$u = a \sin\left(\frac{\pi x}{L}\right)$$

from which it is released at time $t = 0$. Find the displacement $u(x, t)$ from one end. [8]

5. (*a*) Solve the following system of equations by Gauss elimination method : [9]

$$2x_1 + x_2 + x_3 = 10$$

$$3x_1 + 2x_2 + 3x_3 = 18$$

$$x_1 + 4x_2 + 9x_3 = 16.$$

(*b*) Use Runge-Kutta method of fourth order to solve : [8]

$$\frac{dy}{dx} = \sqrt{x+y}, \ y(0) = 1,$$

to find y at $x = 0.1$, taking $h = 0.1$.

Or

6. (*a*) Solve the following system of equations by Gauss-Seidel iteration method : [9]

$$20x + y - 2z = 17$$

$$3x + 20y - z = -18$$

$$2x + 3y + 20z = 25.$$

(*b*) Solve the equation :

$$\frac{dy}{dx} = x^2 + y, \ y(0) = 1$$

to find y at $x = 0.1$, using Euler's modified method taking $h = 0.05$. [8]

SECTION II

7. (*a*) Find the first four moments about mean if the moments about working mean 44.5 of a distribution are –0.4, 2.99, –0.08 and 27.63. Also calculate β_1 and β_2. [6]

(*b*) Find the correlation co-efficient for the data : [6]

x_1	y_1
10	18
14	12
18	24
22	6
26	30
30	36

(*c*) In a male population of 1000, the mean height is 68.16 inches and standard deviation is 3.2 inches. How many men will be expected to be more than 6 feet (72 inches) ?

Given : A(z = 1.2) = 0.3849. [5]

Or

8. (*a*) The scores obtained by two batsmen A and B in 10 matches are given below. Determine who is more consistent and who is better run getter. [7]

Batsman A	**Batsman B**
30	34
44	46
66	70
62	38
60	55
34	48
80	60
46	34
20	45
38	30

(*b*) Assume that, the probability of an individual coal miner being killed in a mine accident during a year is $\frac{1}{2400}$. Calculate the probability that in mine employing 200 miners, there will be at least one will be killed by accident in a year. [5]

(*c*) A dice is thrown 6 times. If "getting an odd number" is a "success", what is the probability of ?

(*i*) 5 success

(*ii*) At least 5 success. [5]

9. (*a*) If

$$\bar{r} \cdot \frac{d\bar{r}}{dt} = 0$$

then show $\bar{r}$ have constant magnitude. [5]

(*b*) Find directional derivative of :

$$\phi = x^2y^2 + y^2z^2 + z^2x^2 \text{ at } (1, 1, -2)$$

in the direction of tangent to the curve

$x = e^t$; $y = 2\sin t - 1$; $z = t - \cos t$ at $t = 0$. [6]

(*c*) Find the value of m if

$$\overline{\text{F}} = (x + 2y)\,\hat{i} + (my + 4z)\,\hat{j} + (5z + 6x)\,\hat{k}$$

is solenoidal. [5]

Or

10. (*a*) Prove the following identities (any *two*) : [8]

(*i*) $\nabla^4\left(r^2 \log r\right) = \dfrac{6}{r^2}$

(*ii*) $\nabla \cdot \left(\dfrac{\bar{a} \times \bar{r}}{r^n}\right) = 0$

(*iii*) $\nabla^2 f(r) = f''(r) + \dfrac{2}{r} f'(r)$.

(*b*) Find constants a, b, c so that :

$$\overline{\text{F}} = (x + 2y + az)\,\hat{i} + (bx - 3y - z)\,\hat{j} + (4x + cy + 2z)\,\hat{k}$$

is irrotational. [4]

(*c*) Find the values of a, b, c so that the directional derivative of :

$$\phi = axy^2 + byz + cz^2x^2$$

at (2, 1, 1) has a maximum magnitude 12 in the direction parallel to x-axis. [4]

11. (*a*) If

$$\overline{\mathrm{F}} = \left(3x^2 + 6y\right)\hat{i} - 14yz\,\hat{j} + 20xz^2\,\hat{k}$$

then find work done of $\overline{\mathrm{F}}$ from (0, 0, 0) to (1, 1, 1) along the path C : [5]

$$x = t;\ y = t^2;\ z = t^3.$$

(*b*) By using Stokes' theorem, evaluate :

$$\iint_{\mathrm{S}} \nabla \times \overline{\mathrm{F}} \,.\, d\overline{s}$$

where,

$$\overline{\mathrm{F}} = (2y + x)\,\hat{i} + (x - y)\,\hat{j} + (z - x)\,\hat{k}$$

and S is the surface of the region bounded by

$$x = 0,\ y = 0 \text{ and } x + y + z = 1$$

which is not included in xoy plane. [6]

(*c*) Use Divergence theorem, evaluate :

$$\iint \overline{\mathrm{F}} \,.\, d\overline{s},$$

where

$$\overline{\mathrm{F}} = x\hat{i} + y\hat{j} + z\hat{k} \text{ and}$$

$$d\overline{s} = dy\, dz\, \hat{i} + dz\, dx\, \hat{j} + dx\, dy\, \hat{k},$$

over the surface of sphere of radius a. [6]

Or

12. (*a*) Using Green's theorem, evaluate :

$$\int_{\mathrm{C}} \left(xy - x^2\right) dx + x^2\, dy$$

along the curve C formed by : [6]

$$y = 0,\ x = 1,\ y = x.$$

(*b*) Evaluate :

$$\iint_{\mathrm{S}} \left(\nabla \times \overline{\mathrm{F}}\right) . \hat{n}\, ds$$

where

$$\overline{\mathrm{F}} = (x - y)\,\hat{i} + \left(x^2 + yz\right)\hat{j} - 3xy^2\,\hat{k}$$

and S is surface of cone

$$z = 4 - \sqrt{x^2 + y^2}$$

above xoy plane. [6]

(*c*) Show that :

$$u = 3x + 2y;$$

$$v = 2x - 3y;$$

$$w = 0$$

are the velocity components of possible fluid motion. Determine whether flow is irrotational. [5]