

Total No. of Questions : 12]

[Total No. of Printed Pages : 7

[3861]-151

F. E. (Semester - I) Examination - 2010

ENGINEERING MATHEMATICS - I

(2008 Pattern)

Time : 3 Hours]

[Max. Marks : 100

Instructions :

- (1) *Answers to the two sections should be written in separate answer books.*
 - (2) *Black figures to the right indicate full marks.*
 - (3) *Neat diagrams must be drawn wherever necessary.*
 - (4) *Assume suitable data, if necessary.*
 - (5) *Use of electronic pocket calculator is allowed.*
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SECTION - I

Q.1) (A) Reduce the following matrix A to its normal form and hence find its rank, where **[05]**

$$A = \begin{bmatrix} 2 & -1 & 1 & 3 \\ 2 & 4 & -1 & 0 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

(B) Examine the consistency of the system of the following equations. If consistent, solve system of the equations : **[06]**

$$x + y - z + t = 2$$

$$2x + 3y + 4t = 9$$

$$y - 2z + 3t = 2$$

- (C) Verify Cayley Hamilton Theorem for the matrix [07]

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

OR

- Q.2) (A) Find Eigen Values and corresponding Eigen Vectors for the matrix [07]

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

- (B) Examine whether the following vectors are linearly dependent. If so, find the relation between them :

$$X_1 = (2, -2, 4), X_2 = (-1, 3, -3), X_3 = (1, 1, 1) \quad [05]$$

- (C) Find values of a, b, c so that the matrix

$$A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$$

becomes an orthogonal matrix. [06]

- Q.3) (A) If $\frac{Z-1}{Z+i}$ is a purely imaginary number, then show that the locus of Z is a circle. [06]

- (B) Show that the continued product of all values of $(1 + i\sqrt{3})^{\frac{1}{4}}$

$$\text{is } 2 \left[\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right] \quad [05]$$

- (C) If α, β are roots of an equation,
 $\sin^2\theta z^2 - (2\sin\theta\cos\theta)z + 1 = 0$, prove that
 $\alpha^n + \beta^n = 2\cos n\theta \operatorname{cosec}^n\theta$, where n is an integer. [05]

OR

- Q.4)** (A) Find $\tanh x$ if $5 \sinh x - \cosh x = 5$ [05]
 (B) If $u + iv = \sin(x + iy)$,
 prove that :
 (a) $u^2 \operatorname{cosec}^2 x - v^2 \sec^2 x = 1$
 (b) $u^2 \operatorname{sech}^2 x + v^2 \operatorname{cosech}^2 x = 1$ [05]
 (C) A square lies above real axis in Argand's diagram and has two of its vertices at origin and the point $3 + 2i$. Find the rest two vertices of the square. [06]

- Q.5)** (A) If $y = \frac{x^3}{x^2 - 1}$,
 then find n^{th} order differential coefficient of y w.r. to x . [05]
 (B) If $y = \sin^{-1} [3x - 4x^3]$,
 prove that $(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - n^2 y_n = 0$. [05]
 (C) Test convergence of the series : **(Any One)** [06]
 (a) $1 + \frac{1}{2} + \frac{1 \cdot 3}{2 \cdot 4} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} + \dots$
 (b) $\sum_{n=1}^{\infty} \frac{(n+1)x^n}{n^2}$

OR

Q.6) (A) If $y = (2x + 1) \log (4x + 3)$,
then find y_{20} . **[05]**

(B) If $y = \left[x + \sqrt{x^2 - 1} \right]^m$,
prove that $(x^2 - 1) y_{n+2} + (2n + 1) xy_{n+1} + (n^2 - m^2) y_n = 0$. **[05]**

(C) Test convergence of the series : **(Any One)** **[06]**

(a) $\frac{1}{1^2 + m} + \frac{2}{2^2 + m} + \frac{3}{3^2 + m} + \dots$

(b) $\sum_{n=1}^{\infty} \frac{4.7.10 \dots (3n+1)}{1.2.3.4 \dots n}$

SECTION - II

Q.7) (A) Expand $\frac{x}{e^x - 1}$ upto x^4 . **[05]**

(B) Use Taylor's Theorem to obtain approximate value of $\sqrt{10}$ to
four decimal places. **[05]**

(C) Solve : **(Any One)** **[06]**

(a) Find a and b, if

$$\lim_{x \rightarrow 0} \frac{a \sin^2 x + b \log \cos x}{x^4} = -\frac{1}{2}$$

(b) Evaluate $\lim_{x \rightarrow 0} \left(\sin^2 \frac{\pi}{2 - ax} \right)^{\sec^2 \frac{\pi}{2 - bx}}$

OR

Q.8) (A) Expand $\sin^{-1}x$ in ascending powers of x . **[05]**

(B) Expand $3x^2 - 2x^2 + x - 4$ in powers of $(x + 2)$. **[05]**

(C) Solve : (Any One) **[06]**

(a) Evaluate $\lim_{x \rightarrow 0} \frac{\tan x - 2\sin x + x}{x^5}$

(b) Evaluate $\lim_{x \rightarrow 0} \frac{e^x \sin x - x - x^2}{x^2 + x \log(1-x)}$

Q.9) Solve : (Any Two) **[16]**

(A) Find value of n so that $u = r^n(3\cos^2 \theta - 1)$

satisfies $\frac{\partial}{\partial r} \left(r^2 \frac{\partial u}{\partial r} \right) + \frac{1}{\sin \theta} \cdot \left(\sin \theta \frac{\partial u}{\partial \theta} \right)$.

(B) If $ux + vy = 0$; $\frac{u}{x} + \frac{v}{y} = 1$,

show that : $\left(\frac{\partial v}{\partial y} \right)_x - \left(\frac{\partial u}{\partial x} \right)_y = \frac{x^2 + y^2}{x^2 - y^2}$

(C) If $x = e^u \operatorname{cosec} v$; $y = e^u \cot v$, then

show that : $\left(\frac{\partial z}{\partial x} \right)^2 - \left(\frac{\partial z}{\partial y} \right)^2 = e^{-2u} \left[\left(\frac{\partial z}{\partial u} \right)^2 - \sin^2 v \left(\frac{\partial z}{\partial v} \right)^2 \right]$

OR

Q.10) Solve : (Any Two)

[16]

(A) If $u = (2x + 3y)^n + \frac{1}{(x - y)^n}$,

show that : $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} + xu_x + yu_y = n^2 u$.

(B) If $x^3 + y^3 - 3axy = 0$,

show that : $\frac{d^2 y}{dx^2} + \frac{2a^3 xy}{(y^2 - ax)^3} = 0$.

(C) If $f(xy^2, z - 2x) = 0$,

prove that : $x \frac{\partial z}{\partial x} - \frac{y}{2} \frac{\partial z}{\partial y} = 2x$.

Q.11) (A) The area of ΔABC is calculated using the formula

$\Delta = \frac{1}{2} ab \sin C$. Errors of 2%, 3%, 4% are made in measuring a, b, C respectively. If the correct value of C is 30° , find % error in the calculated value of Δ .

[06]

(B) If $x = u + v$; $y = v^2 + w^2$; $z = w^3 + u^3$,

show that : $\frac{\partial u}{\partial x} = \frac{vw}{vw + u^2}$.

[06]

(C) Find Stationary values of $u = x + y + z$ if $xy + yz + zx = 3a^2$. **[06]**

OR

Q.12) (A) Verify $JJ' = 1$ for $x = e^u \tan v$ and $y = e^u \sec v$. **[06]**

(B) Examine for functional dependence/independence. If dependent, find relation between them :

$$u = \frac{x - y}{x + a} \quad ; \quad v = \frac{x + a}{y + a} \quad \text{[06]}$$

(C) The sum of three positive numbers is 'a'. Determine maximum value of their product. **[06]**