Total No. of Questions—8] [Total No. of Printed Pages—7

Seat No.	

[4757]-1031

S.E. (Electrical/Instru.) (First Semester)

EXAMINATION, 2015

ENGINEERING MATHEMATICS—III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

N.B. **:—** (*i*) Attempt Q. No. **1** *or* Q. No. **2,** Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6,** Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of non-programable electronic pocket calculator and steam tables is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = e^{e^x}$

(*ii*) $x^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + 4y = \cos(\log x) + x\sin(\log x)$

(*iii*) $(D^2 + 4)\, y = \sec 2x$ by variation of parameters method.

(*b*) Find Laplace-transform of $\frac{1-\cos t}{t}$. [4]

Or

2. (*a*) The charge Q on the plate of condenser satisfies the equation : [4]

$$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = \frac{E}{L}\sin\frac{t}{\sqrt{LC}}$$

Prove that the charge at any time t is given by

$$Q = \frac{EC}{2}\left[\sin \omega t - \omega t \cos \omega t\right]$$

where $\omega = \frac{1}{\sqrt{LC}}$ and $Q = 0$ at $t = 0$.

(*b*) Solve (any *one*) : [4]

(*i*) $L[t \cup(t - 4) - t^3 \delta(t - 2)]$

(*ii*) $L^{-1}\left[\frac{1}{s^2(s+1)^2}\right]$ by convolution theorem.

(*c*) Solve by Laplace-transform method : [4]

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t$$

with $y(0) = 0$ and $y'(0) = 1$.

3. (*a*) Find inverse Fourier sine transform of : [4]

$$F_s(\lambda) = \frac{1}{\lambda} e^{-a\lambda}, \ \lambda > 0 .$$

(*b*) Find inverse *z*-transform of : [4]

$$F(z) = \frac{z^2}{(z-1)\left(z-\frac{1}{2}\right)^2}, |z| > \frac{1}{2}.$$

(*c*) Find directional derivatives of

$$\phi = e^{2x-y-z}$$

at (1, 1, 1) in the direction of tangent to curve

$$x = e^{-t}, \ y = 2\sin t + 1, \ z = t - \cos t$$

at $t = 0$. [4]

Or

4. (*a*) Prove the following (any *one*) : [4]

(*i*) $\nabla \cdot \left[r \, \nabla \left(\frac{1}{r^4} \right) \right] = \frac{8}{r^5}$

(*ii*) $\nabla \cdot \left(\frac{\bar{a} \times \bar{r}}{r} \right) = 0$.

(*b*) Show that vector field [4]

$$\bar{F} = (6xy + z^3)\,\bar{i} + (3x^2 - z)\,\bar{j} + (3xz^2 - y)\bar{k}$$

is irrotational. Find scalar function ϕ such that :

$$\bar{F} = \nabla\phi .$$

(*c*) Solve the difference equation : [4]

$$f(k + 2) + 3\, f(k + 1) + 2\, f(k) = 0,$$

$$f(0) = 0,\ f(1) = 1.$$

5. (*a*) Evaluate :

$$\int_C \overline{F} \cdot d\overline{r}$$

where

$$\overline{F} = zi + xj + yk$$

and C is the arc of the curve $x = \cos t$, $y = \sin t$, $z = t$ from $t = 0$ to $t = 2\pi$. [4]

(*b*) Evaluate :

$$\iint_S \nabla \times \overline{F} \cdot d\overline{s}$$

for vector field

$$\overline{F} = 4yi - 4xj + 3k$$

where s is a disc of radius 1 lying on the plane $z = 1$ and C is its boundary. [4]

(*c*) Evaluate :

$$\iint_S \left(x^3 \; dydz + x^2y \; dzdx + x^2z \; dxdy\right)$$

where S is the closed surface consisting of the circular cylinder

$$x^2 + y^2 = a^2,$$

$z = 0$ and $z = b$. [5]

Or

6. (*a*) Using Green's theorem, evaluate

$$\int_C \left(\frac{1}{y}dx + \frac{1}{x}dy\right)$$

where C is the boundary of the region bounded by the parabola $y = \sqrt{x}$ and the lines $x = 1$, $x = 4$. [4]

(*b*) Evaluate :

$$\iint_S \overline{F} \, . \, d\overline{s}$$

using Gauss divergence theorem where

$$\overline{F} = 2xyi + yz^2j + xzk$$

and s is the region bounded by

$x = 0$, $y = 0$, $z = 0$, $y = 3$, $x + 2z = 6$. [5]

(*c*) Evaluate :

$$\int_C \overline{F} \cdot d\overline{r}$$

by Stokes' theorem, where

$$\overline{F} = y^2 i + x^2 j - (x + z)k$$

and C is the boundary of the triangle with vertices at (0, 0, 0), (1, 0, 0) and (1, 1, 0). [4]

7. (*a*) If $\phi + i\psi$ is complex potential for an electric field and

$$\phi = -2xy + \frac{x}{x^2 + y^2},$$

find function ψ. [4]

(*b*) Evaluate :

$$\oint_C \frac{z + 4}{(z+1)^2 (z+2)^2} dz$$

where 'C' is a circle $|z + 1| = \frac{1}{2}$. [5]

(*c*) Find the bilinear transformation which maps points 1, 0, i of z-plane onto the points ∞, –2, $-\frac{1}{2}(1 + i)$ of w-plane. [4]

Or

8. (*a*) Show that analytic function with constant amplitude is constant. [4]

(*b*) Evaluate :

$$\int_{2+4i}^{5-5i} (z+1)\, dz$$

along the line joining the points $2 + 4i$ and $5 - 5i$. [5]

(*c*) Find the image of Hyperbola :

$$x^2 - y^2 = 1$$

under the transformation $w = \frac{1}{z}$. [4]