Total No. of Questions—**8**] [Total No. of Printed Pages—**5**

Seat No.	

[5152]-105

S.E. (Civil) (I Sem.) EXAMINATION, 2017

ENGINEERING MATHEMATICS—III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Neat diagrams must be drawn wherever necessary.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(*iv*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* of the following : [8]

(*i*) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = 2\cos x + 3x + 2 + 3e^x$

(*ii*) $x^3\dfrac{d^3y}{dx^3} + 2x^2\dfrac{d^2y}{dx^2} + 2y = 10\left(x + \dfrac{1}{x}\right).$

(*iii*) Use the method of variation parameters to solve the linear differential equation :

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}.$$

(*b*) Solve the following system of linear equations by Gauss elimination method : [4]

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 2y + 9z = 34.$$

P.T.O.

Or

2. (*a*) Apply Runge-Kutta method of 4th order to solve the differential equation : [4]

$$\frac{dy}{dx} = \sqrt{x + y}, \ y(0) = 1$$

to find $y(0.2)$ with $h = 0.2$.

(*b*) Solve the system of simultaneous symmetric equations : [4]

$$\frac{dx}{3z - 4y} = \frac{dy}{4x - 2z} = \frac{dz}{2y - 3x}.$$

(*c*) Solve the following system of equations of Cholesky method : [4]

$$4x + 2y + 14z = 14$$
$$2x + 17y - 5z = -101$$
$$14x - 5y + 83z = 155.$$

3. (*a*) Obtain correlation coefficient between population density (per square miles) and death rate (per thousand persons) from data related to 5 cities : [4]

Population Density	**Death Rate**
200	12
500	18
400	16
700	21
800	10

(*b*) In a certain factory turning out razor blades, there is a small chance of 1/500 for any blade to be defective. The blades are supplied in a packet of 10. Use Poisson distribution to calculate the approximate number of packets containing no defective and two defective blades, in a consignment of 10,000 packets. [4]

(*c*) If directional derivative of $\phi = ax^2y + by^2z + cz^2x$ at (1, 1, 1) has maximum manitude 15 in the direction parallel to $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$, hence find the values of *a*, *b*, *c*. [4]

Or

4. (*a*) The first four moments of a distribution about *x* = 2 are 1, 2.5, 5.5 and 1.6. Calculate first four moments about mean. Also find β_1 and β_2. [4]

(*b*) Prove the following (any *one*) : [4]

(*i*) $\nabla \times \left(\frac{\overline{a} \times \overline{r}}{r^3}\right) = -\frac{\overline{a}}{r^3} + \frac{3(\overline{a} \cdot \overline{r})}{r^5}\overline{r}$

(*ii*) $\nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r}\frac{df}{dr}$.

(*c*) If the vector field $\overline{F} = (x + 2y + az)\overline{i} + (bx - 3y - z)\overline{j} + (4x + cy + 2z)\overline{k}$ is irrotational, find *a*, *b*, *c* and determine ϕ such that $\overline{F} = \nabla\phi$. [4]

5. (*a*) Evaluate $\int_C \overline{F} \cdot d\overline{r}$ for $\overline{F} = (2y+3)\overline{i} + xz\overline{j} + (yz - x)\overline{k}$ along the straight line joining (0, 0, 0) and (3, 1, 1). [4]

(*b*) Evaluate $\iint_S (\nabla \times \overline{F}) \cdot d\overline{S}$ for $\overline{F} = -y^3\overline{i} + x^3\vec{j}$ and closed curve 'C' is boundary of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. [4]

(*c*) Use Gauss-divergence theorem to evaluate :

$\iint_S \left(x^3\vec{i} + y^3\vec{j} + z^3\overline{k}\right) \cdot d\overline{S}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 9$. [5]

Or

6. (*a*) Evaluate $\int \overline{F} \cdot d\overline{r}$ using Green's theorem where,

$\overline{F} = (2x^2 - y^2)\vec{i} + (x^2 - y^2)\vec{j}$ and C is the circle $x^2 + y^2 = 1$ above x-axis. [4]

(*b*) Using Stokes' theorem evaluate $\int_C \overline{F} \cdot d\overline{r}$ where

$\overline{F} = y^2\vec{i} + x^2\vec{j} - (x + z)\vec{k}$ and 'C' is the boundary of triangle with vertices (0, 0, 0), (1, 0, 0) and (1, 1, 0). [4]

(*c*) Prove that $\iiint_V \frac{dV}{r^2} = \iint_S \frac{\overline{r} \cdot \hat{n}}{r^2} dS$ where, V is the volume bounded by closed surface S. [5]

7. (*a*) Solve the wave equation $\frac{\partial^2 y}{\partial t^2} + C^2 \frac{\partial^2 y}{\partial x^2}$ subjected to the conditions : [6]

(*i*) $y(0, t) = 0$ for all t

(*ii*) $y(l, t) = 0$ for all t

(*iii*) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$

(*iv*) $y(x, 0) = k(lx - x^2)$, for $0 \leq x \leq l$.

(*b*) An infinitely long uniorm metal plate is enclosed between the lines $y = 0$ to $y = l$ for $(x > 0)$. The temperature is zero along the sides $y = 0$, $y = l$ and at infinite end. If the edge $x = 0$ kept at constant temperature u_0, find temperature distribution $u(x, y)$. [7]

Or

8. (*a*) Solve the one-dimensional heat flow equation : [6]

$$\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$$

subjected to the conditions :

(*i*) $u(x, t)$ bounded

(*ii*) $u(0, t) = 0$ for all t

(*iii*) $u(\pi, t) = 0$ for all t

(*iv*) $u(x, 0) = x, \; 0 < x < \pi$.

(*b*) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subjected to the conditions : [7]

(*i*) $u(0, y) = 0$ for all y

(*ii*) $u(1, y) = 0$ for all y

(*iii*) $u(x, \infty) = 0$, for all x

(*iv*) $u(x, 0) = x(1 - x)$ for $0 < x < 1$.