

Total No. of Questions—8]

[Total No. of Printed Pages—7

Seat No.	
-------------	--

[4757]-1001**S.E. (Civil) (First Semester) EXAMINATION, 2015****ENGINEERING MATHEMATICS III****(2012 PATTERN)****Time : Two Hours****Maximum Marks : 50****N.B.** :— (i) Neat diagrams must be drawn wherever necessary.

(ii) Figures to the right indicate full marks.

(iii) Use of electronic pocket calculator and steam tables is allowed.

(iv) Assume suitable data, if necessary.

1. (a) Solve any *two* of the following : [8]

(i)
$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 4 + 2^x + 3e^{-x} + \cos x$$

(ii)
$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = x + x^{-1}$$

(iii) Use the method of variation of parameters to solve the linear differential equation :

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x .$$

P.T.O.

- (b) Solve the following system of linear equations by Gauss Elimination method : [4]

$$5x - 2y + 2z = 5, \quad 2x + y - z = 2, \quad x - y + z = 1.$$

Or

2. (a) Solve the system of simultaneous symmetric equations : [4]

$$\frac{dx}{2x} = \frac{dy}{-y} = \frac{dz}{4xy^2 - 2z}.$$

- (b) Apply Runge-Kutta method of 4th order to solve the differential equation :

$$\frac{dy}{dx} = x + y^2, \quad y(0) = 1$$

to find y for $0 \leq x \leq 0.2$ with $h = 0.1$. [4]

- (c) Solve the following system of equations by Cholesky method : [4]

$$2x + 3y + z = 0$$

$$x + 2y - z = -2$$

$$-x + y + 2z = 0.$$

3. (a) The first four moments about the value 4 are -1.5 , 17 , -30 and 108 . Calculate the moments about the mean. Also find coefficient of skewness and kurtosis. [4]

(b) Assume the mean height of soldiers to be 68.22 inches with a variance of 10.8 inches. How many soldiers in a regiment of 10,000 would you expect to be over 6 feet tall, where the data is normally distributed. (Given : $\phi(1.15) = 0.3749$) [4]

(c) Find the directional derivative of

$$\phi = xy^2 + yz^3$$

at the point $(2, -1, 1)$ in the direction of vector $i + 2j + 2k$. [4]

Or

4. (a) Attempt any one : [4]

(i) Prove that $\frac{\vec{r}}{r^3}$ is solenoidal.

(ii) Show that :

$$\nabla \cdot \left[r \nabla \frac{1}{r^n} \right] = \frac{n(n-2)}{r^{n+2}}.$$

(b) Verify whether :

$$\vec{F} = (2xyz^2)i + (x^2z^2 + z \cos yz)j + (2x^2yz + y \cos yz)k$$

is irrotational. [4]

(c) Two lines of regression are :

$$5y - 8x + 17 = 0 \quad \text{and} \quad 2y - 5x + 14 = 0.$$

If $\sigma_y^2 = 16$, find : [4]

(i) σ_x^2

(ii) Coefficient of correlation.

5. (a) Evaluate $\int_C \bar{F} \cdot d\bar{r}$ for

$$\bar{F} = (2xy + 3x^2)\bar{i} + (x^2 + 4yz)\bar{j} + (2y^2 + 6yz)\bar{k}$$

where C is the curve $x = t$, $y = t^2$, $z = t^3$ joining (0, 0, 0) and (1, 1, 1). [4]

(b) Use divergence theorem to evaluate $\iiint_S \bar{F} \cdot d\bar{s}$

for $\bar{F} = 4xz\bar{i} - y^2\bar{j} + yz\bar{k}$ over the surface of cube bounded by the planes $x = 0$, $x = 2$, $y = 0$, $y = 2$, $z = 0$, $z = 2$. [4]

(c) Using Stokes' theorem evaluate $\iint_S (\nabla \times \bar{F}) \cdot d\bar{s}$

for $\bar{F} = (x^3 - y^3)\bar{i} - xyz\bar{j} + y^3\bar{k}$ where S is the surface $x^2 + ay^2 + z^2 - 2x = 4$ above the plane $x = 0$. [5]

Or

6. (a) Using Green's theorem, evaluate $\oint \bar{F} \cdot d\bar{r}$ for the field :

$$\bar{F} = x^2 \bar{i} + xy \bar{j}$$

over the region R enclosed by $y = x^2$ and then line $y = x$. [4]

- (b) Use divergence theorem to evaluate :

$$\iiint_S (x^3 \bar{i} + y^3 \bar{j} + z^3 \bar{k}) \cdot d\bar{s}$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 16$. [4]

- (c) Evaluate $\int_C \bar{F} \cdot d\bar{r}$ using Stokes' theorem for :

$$\bar{F} = 4y \bar{i} + 2z \bar{j} + 6y \bar{k}$$

where C is the intersection of :

$$x^2 + y^2 + z^2 = 2z, \quad x = z - 1. \quad [5]$$

7. (a) A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = a \sin \frac{\pi x}{l}$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by : [7]

$$y(x, t) = a \sin \left(\frac{\pi x}{l} \right) \cos \left(\frac{\pi ct}{l} \right).$$

(b) Solve :

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$$

subject to : [6]

(i) $u(0, t) = 0$

(ii) $u_x(l, t) = 0$

(iii) $u(x, t)$ is bounded and

(iv) $u(x, 0) = \frac{u_0 x}{l}, 0 \leq x \leq l.$

Or

8. (a) Solve the equation :

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

subject to : [7]

(i) $v = 0$ when $y \rightarrow \infty$ for all x

(ii) $v = 0$ when $x = 0$ for all y

(iii) $v = 0$ when $x = l$ for all y

(iv) $v = x(l - x)$ when $y = 0$ for $0 < x < l$.

(b) Solve the wave equation :

[6]

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

under the conditions :

(i) $u(0, t) = 0$

(ii) $u(\pi, t) = 0$

(iii) $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$

(iv) $u(x, 0) = x, \quad 0 < x < \pi.$

www.sppuonline.com