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[5057]-201**S.E. (Civil) (First Semester) EXAMINATION, 2016****ENGINEERING MATHEMATICS-III****(2012 PATTERN)****Time : Two Hours****Maximum Marks : 50**

N.B. :- (i) Answer Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4,
Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.

(ii) Figures to the right indicate full marks.

(iii) Non-programmable electronic pocket calculator is allowed.

(iv) Assume suitable data, if necessary.

(v) Neat diagrams must be drawn wherever necessary.

1. (a) Solve any two of the following : [8]

(i) $(D^2 - 4D - 4)y = e^{2x} \sin 3x$

(ii) $(x^2 D^2 - 2x D - 4) y = x^2 + 2 \log x$

(iii) $(D^2 - 2D + 2) y = e^x \tan x$ (using method of variation of parameters)

P.T.O.

(b) Solve the following system of equations : [4]

$$\left. \begin{array}{l} 28x + 4y - z = 32 \\ 2x + 17y + 4z = 35 \\ x + 3y + 10z = 24 \end{array} \right\} \text{ by Gauss-Seidel method.}$$

Or

2. (a) A 3N weight stretches a spring '15 cm'. If the weight is pulled '10 cm' below the equilibrium position and then released. Find the displacement function at any time ' t '. [4]

(b) Solve the following system of equation of Cholesky's method : [4]

$$2x_1 - x_2 = 1, -x_1 + 3x_2 + x_3 = 0 \quad \text{and} \quad x_1 + 2x_3 = 0$$

(c) Solve the following equation :

$$\frac{dy}{dx} = x - 2y,$$

using Runge-Kutta fourth order method, given that $y = 1$ when $x = 0$ and find y at $x = 0.1$ taking $h = 0.1$. [4]

3. (a) The first four moments of a distribution about the values of 5 are 2, 20, 40 and 50. From the given information obtain the first four central moments, mean, standard deviation and coefficient of skewness and kurtosis. [4]

(b) In a sample of 1000 cases, the mean of a certain test is '14' and standard deviation is 2.5. Assuming the distribution to be normal, find how many students score between 12 and 15 given that $p(z > 0.4) = 0.1554$ and $p(z < -0.8) = 0.2881$. [4]

(c) Show that :

$$\bar{F} = (y \cdot e^{xy} \cdot \cos z) \hat{i} + (x \cdot e^{xy} \cdot \cos z) \hat{j} - (e^{xy} \cdot \sin z) \hat{k}$$

is irrotational. [4]

Or

4. (a) Attempt any one : [4]

(i) Show that :

$$\nabla f(r) = f'(r) \frac{\bar{r}}{r}$$

where

$$\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

(ii) Show that :

$$\nabla^2 \left[\frac{1}{r} \log r \right] = \frac{-1}{r^3}$$

(b) Find the directional derivative of : [4]

$$\phi = x^2 - y^2 - 2z^2$$

at the point P(2, -1, 3) in the direction of PQ where Q is (5, 6, 4).

(c) Given :

$$n = 6, \Sigma(x - 18.5) = -3, \Sigma(y - 50) = 0, \Sigma(x - 18.5)^2 = 19,$$

$$\Sigma(y - 50)^2 = 850, \Sigma(x - 18.5)(y - 50) = -120,$$

calculate the coefficient of correlation. [4]

5. (a) Find the work done by :

$$\vec{F} = 2xy^2 \hat{i} + (2x^2y + y) \hat{j} + xz^2 \hat{k},$$

in moving a particle from the point (0, 0, 0) to the point (2, 4, 0) along the curve $y = x^2, z = 0$. [4]

- (b) Evaluate :

$$\iint_S \vec{F} \cdot d\vec{S}$$

where $\vec{F} = \frac{\vec{r}}{r^2}$ and 'S' is the surface of the sphere

$$x^2 + y^2 + z^2 = 4. \quad [5]$$

- (c) Evaluate : [4]

$$\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$$

where

$$\vec{F} = xy^2 \hat{i} + y \hat{j} + xz^2 \hat{k},$$

and 'S' is the rectangular surface bounded by :

$$x = 0, y = 0, x = 1, y = 2, z = 0.$$

Or

6. (a) Evaluate : [4]

$$\int_C (\sin y - y^3) dx + (xy^2 + x \cos y) dy,$$

using Green's theorem where 'C' is the circle :

$$x^2 + y^2 = a^2.$$

- (b) Use Gauss's Divergence theorem to evaluate : [5]

$$\iint_S \bar{F} \cdot d\bar{S}$$

where

$$\bar{F} = yz\hat{i} + xz\hat{j} + xy\hat{k}$$

and S is the surface of sphere :

$$x^2 + y^2 + z^2 = 1$$

in the first octant.

- (c) Prove that : [4]

$$\int_C (\bar{a} \times \bar{r}) \cdot d\bar{r} = 2 \iint_S \bar{a} \cdot d\bar{S}$$

7. (a) An elastic string is stretched between two points at a distance 'l' apart. In its equilibrium position, A point at a distance 'a' ($a < l$) from one end is displaced through the distance 'b' transversely and then released from this position. Obtain $y(x, t)$ the vertical displacement if : [7]

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

- (b) Solve : [6]

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$$

if :

(i) $u(0, t) = 0$

$$(ii) u_x(l, t) = 0$$

(iii) $u(x, t)$ is bounded and

$$(iv) u(x, 0) = \frac{u_0 \cdot x}{l}, \quad 0 \leq x \leq l$$

Or

8. (a) A tightly stretched string of length ' l ' is initially in equilibrium position is set vibrating by giving to each of its points, the velocity : [7]

$$\left(\frac{\partial y}{\partial t} \right)_{t=0} = v_0 \sin^3 \left(\frac{\pi x}{l} \right).$$

Find $y(x, t)$ using :

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}.$$

- (b) An infinitely long uniform metal plate is enclosed between lines $y = 0$ and $y = l$ for $x > 0$. The temperature is zero along the edges $y = 0$, $y = l$ and at infinity. If edge $x = 0$ is kept at a constant temperature ' u_0 ', find the temperature distribution $u(x, y)$. [6]