

Total No. of Questions—8]

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[4857]-1005**S.E. (Civil) (First Semester) EXAMINATION, 2015****ENGINEERING MATHEMATICS-III****(2012 PATTERN)****Time : Two Hours****Maximum Marks : 50**

- N.B.** :— (i) Answer Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6 and Q. No. 7 or Q. No. 8
- (ii) Neat diagrams must be drawn wherever necessary.
- (iii) Figures to the right indicate full marks.
- (iv) Use of non-programmable electronic pocket calculator is allowed.
- (v) Assume suitable data, if necessary.

1. (a) Solve any two : [8]

(i) $(D^2 - 9D + 18)y = e^{-3x}$

(ii) $(D^3 + 3D^2 - 4)y = x^2 + x + 1$

(iii) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = \sin(2 \log x)$.

(b) Solve the following system of linear equations by Gauss-Seidel method : [4]

$$28x_1 + 4x_2 - x_3 = 32$$

$$2x_1 + 17x_2 + 4x_3 = 35$$

$$x_1 + 3x_2 + 10x_3 = 24$$

P.T.O.

Or

2. (a) Solve the following system of symmetrical simultaneous equation : [4]

$$\frac{dx}{1} = \frac{dy}{3} = \frac{dz}{5z + \tan(y - 3x)}$$

- (b) Use Runge-Kutta method of fourth order to obtain the numerical solution of :

$$\frac{dy}{dx} = x^2 + y^2, y(1) = 1.5$$

in the interval (1, 1.2) with $h = 0.2$. [4]

- (c) Solve the following system by Cholesky method : [4]

$$4x_1 - 2x_2 = 0$$

$$-2x_1 + 4x_2 - x_3 = 1$$

$$-x_2 + 4x_3 = 0$$

3. (a) The first four moments of a distribution about the value 5 are 3, 30, 50 and 60. Obtain the first four central moments and coefficient of skewness and kurtosis. [4]

- (b) Given a normal distribution with $\mu = 50$ and $\sigma = 10$, find the probability that x assumes a value between 45 and 62.

(Given that : $P(z > 0.5) = 0.3085$, $P(z < 1.2) = 0.8849$) [4]

- (c) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ in the direction $2\bar{i} - 3\bar{j} + 6\bar{k}$. [4]

Or

4. (a) Attempt any one : [4]

(i) Show that :

$$\nabla^2 \left[\nabla \cdot \left(\frac{\bar{r}}{r} \right) \right] = 0$$

(ii) If \bar{u} and \bar{v} are irrotational vectors then prove that $\bar{u} \times \bar{v}$ is solenoidal vector.

(b) Verify whether the following field is irrotational : [4]

$$\bar{F} = (6xy + z^3) \hat{i} + (3x^2 - z) \hat{j} + (3xz^2 - y) \hat{k}.$$

(c) Calculate the coefficient of correlation from the following data : [4]

$$n = 20, \Sigma x = 40, \Sigma x^2 = 190, \Sigma y^2 = 200, \Sigma xy = 150, \Sigma y = 40.$$

5. (a) Prove that :

$$\bar{F} = (4xy - 3x^2z^2) \hat{i} + 2x^2z \hat{j} - 2x^3z \hat{k}$$

is a conservative field and also find the work done in moving an object in this field from (0, 0, 0) to (1, 1, 1). [5]

(b) Use divergence theorem to evaluate :

$$\iint_S \bar{F} \cdot d\bar{S}, \text{ where } \bar{F} = x^3 \hat{i} + x^2y \hat{j} + x^2z \hat{k}$$

and S is the surface bounding the region $x^2 + y^2 = a^2$, $z = 0$ and $z = b$. [4]

(c) Evaluate : [4]

$$\iint_S \nabla \times \bar{F} \cdot d\bar{S} \text{ for } \bar{F} = y\hat{i} + z\hat{j} + x\hat{k}$$

where S is the surface of the paraboloid $z = 1 - x^2 - y^2$, $z \geq 0$.

Or

6. (a) Use Green's Lemma to evaluate the line integral : [5]

$$\oint_C (\cos x \sin y - 4y) dx + \sin x \cos y dy,$$

where C is the circle $x^2 + y^2 = 1$.

- (b) Evaluate :

$$\iiint_S (3x dy dz - 2y dz dx + 2z dx dy)$$

over the surface of a sphere of radius a . [4]

- (c) Evaluate : [4]

$$\iint_S \nabla \times \bar{F} \cdot \hat{n} ds$$

for the surface of the paraboloid :

$$z = 4 - x^2 - y^2 \quad (z \geq 0) \quad \text{and} \quad \bar{F} = y^2 \hat{i} + z \hat{j} + xy \hat{k}.$$

7. (a) Solve the equation : [7]

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

with conditions :

(i) $u = 0$ when $y = \infty$

(ii) $u = 0$ when $x = 0$

(iii) $u = 0$ when $x = 1$

(iv) $u = x(1 - x)$ when $y = 0$ for $0 \leq x \leq 1$

- (b) Solve $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ if : [6]

(i) $u(0, t) = 0$

(ii) $u(l, t) = 0$

(iii) $u(x, 0) = u_0, 0 \leq x \leq l$

Or

8. (a) A thin sheet of metal, bounded by x -axis and the lines $x = 0$ and $x = 1$ and stretching to infinity in the y direction has its upper and lower faces perfectly insulated and its vertical edges and the edge at infinity are maintained at 0°C , while over the base temperature of 100°C . Find steady state temperature $u(x, y)$. [6]
- (b) A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $u = a \sin \frac{\pi x}{l}$ from which it is released at time $t = 0$. Find the displacement $u(x, t)$ from one end. [7]

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