Total No. of Questions—**8**] [Total No. of Printed Pages—**7**

Seat No.	

[4657]-501

S.E. (Civil) (First Semester)

EXAMINATION, 2014

ENGINEERING MATHEMATICS—III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Answer all the questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $(D^2 + 2D + 1)y = xe^{-x} \cos x$

(*ii*) $(2x + 1)^2 \dfrac{d^2y}{dx^2} - 2(2x + 1) \dfrac{dy}{dx} - 12y = 6x$

(*iii*) Use method of variation of parameters to solve :

$$(D^2 - 2D + 2)y = e^x \tan x.$$

P.T.O.

(*b*) Solve the following system of equations using Gauss-Seidel iteration method : [4]

$$28x_1 + 4x_2 - x_3 = 32$$

$$x_1 + 3x_2 + 10x_3 = 24$$

$$2x_1 + 17x_2 + 4x_3 = 35$$

Or

2. (*a*) Solve the following system of symmetrical simultaneous equations : [4]

$$\frac{dx}{3z - 4y} = \frac{dy}{4x - 2z} = \frac{dz}{3y - 2x}.$$

(*b*) Use Euler's modified method to find the value of y satisfying the equation : [4]

$$\frac{dy}{dx} = \log(x + y), \; y(1) = 2$$

for $x = 1.2$ and $x = 1.4$ correct to three decimal places by taking $h = 0.2$.

(*c*) Solve the following system of equations by Cholesky's method : [4]

$$2x_1 - x_2 = 1$$

$$-x_1 + 3x_2 + x_3 = 0$$

$$x_1 + 2x_3 = 0.$$

3. (*a*) The first four moments of a distribution about $x = 2$ are 1, 2.5, 5.5 and 1.6. Calculate first four moments about mean. Also find β_1 and β_2. [4]

(*b*) Assuming that the probability of an individual coal miner being killed in a mine accidents during a year is $\frac{1}{2400}$. Use Poisson's distribution to calculate the probability that in a mine employing 200 miners there will be at least one fatal accident in a year. [4]

(*c*) Find the directional derivative of $\phi = xy + yz^2$ at $(1, -1, 1)$ towards the point $(2, 1, 2)$. [4]

Or

4. (*a*) Prove the following (any *one*) : [4]

(*i*) $\nabla^2 \left(\frac{\bar{a} \cdot \bar{b}}{r}\right) = 0$

(*ii*) $\nabla \times \left(\frac{\bar{a} \times \bar{r}}{r^3}\right) = \frac{-\bar{a}}{r^3} + \frac{3(\bar{a} \cdot \bar{r})}{r^5} \bar{r}$.

(*b*) Show that : [4]

$$\bar{F} = (2xz^3 + 6y)\,\bar{i} + (6x - 2yz)\,\bar{j} + (3x^2z^2 - y^2)\,\bar{k}$$

is irrotational. Find the scalar potential ϕ such that $\bar{F} = \nabla\phi$.

(*c*) If two lines of regression are :

$$9x + y - \lambda = 0 \text{ and } 4x + y = \mu$$

and the means of x and y are 2 and –3 respectively, find the values of λ and μ, also find coefficient of correlation between x and y. [4]

5. (*a*) Find the work done in moving a particle once round the circle $x^2 + y^2 = 9$ in the xy plane if the force field $\overline{F}$ is given by : [4]

$$\overline{F} = (2x - y - z)\, i + (x + y - z^2)\, j + (3x - 2y + 4z)\, \hat{k}.$$

(*b*) Evaluate :

$$\iint_S \overline{F} \cdot \hat{n}\, dS,$$

where :

$$\overline{F} = (2x + 3z)\, \hat{i} - (xz + y)\, \hat{j} + (y^2 + 2z)\, \hat{k}$$

and S is the surface of the sphere having centre at (3, –1, 2) and radius 3. [4]

(*c*) Evaluate :

$$\iint_S (\nabla \times \overline{F}) \cdot \hat{n}\, dS,$$

where 'S' is the curved surface of the paraboloid :

$$x^2 + y^2 = 2z$$

bounded by the plane $z = 2$, where : [5]

$$\overline{\mathrm{F}} = 3(x - y)\,\hat{i} + 2xz\,\hat{j} + xy\,\hat{k}.$$

Or

6. (*a*) If :

$$\overline{\mathrm{F}} = (2xz^3 + 6y)\,\hat{i} + (6x - 2yz)\,\hat{j} + (3x^2 z^2 - y^2)\,\hat{k}$$

evaluate :

$$\int_{\mathrm{C}} \overline{\mathrm{F}} \cdot d\bar{r}$$

where C is the join of (0, 0, 0) and (1, 1, 1). Is the force $\overline{\mathrm{F}}$ conservative ? [4]

(*b*) Prove that :

$$\iiint_{\mathrm{V}} \frac{1}{r^2}\, dv = \iint \frac{1}{r^2}\,\bar{r} \cdot d\overline{\mathrm{S}}$$

where S is the closed surface enclosing volume V. Hence evaluate :

$$\iint \frac{x\,\hat{i} + y\hat{j} + z\hat{k}}{r^2} \cdot d\overline{\mathrm{S}}$$

where S is the surface of the sphere : [5]

$$x^2 + y^2 + z^2 = a^2.$$

(*c*) Show that the velocity potential :

$$\phi = (x^2 - 2y^2 + z^2)$$

satisfies the Laplace's equation. Also determine the stream-lines. [4]

7. (*a*) A string of length l is stretched and fastened to two ends. Motion is started by displacing the string in the form : [7]

$$u(x) = a \sin\left(\frac{\pi x}{l}\right)$$

from which it is released at $t = 0$.

Find the displacement u at any time 't', if it satisfies the equation : [6]

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}.$$

(*b*) Solve :

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$$

if :

(*i*) $u(x, \infty)$ is finite

(*ii*) $u(0, t) = 0$

(*iii*) $u(l, t) = 0$

(*iv*) $u(x, 0) = \frac{u_0 x}{l}, \quad 0 < x < l$

Or

8. (*a*) An infinitely long plane uniform plate is bounded by two parallel edges in the y direction and an end at right angles to them. The breadth of the plate is π. The end is maintained at temperature u_0 at all points and other edges at zero temperature. Find steady state temperature $u(x, y)$, if it satisfies : [7]

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

(*b*) Use Fourier transform to solve : [6]

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}; \; 0 < x < \infty, \quad t > 0$$

where $u(x, t)$ satisfies the conditions :

(*i*) $|u(x, t)| < M$

(*ii*) $\left(\frac{\partial u}{\partial x}\right)_{x=0} = 0$, at $t > 0$

(*iii*) $u(x, 0) = x, \; 0 < x < 1$
$\qquad\quad = 0, \; x > 1$