Total No. of Questions—8] [Total No. of Printed Pages—6

Seat No.	

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S.E. (Civil Engg.) (I Sem.) EXAMINATION, 2019

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Neat diagrams must be drawn wherever necessary.

(*ii*) Assume suitable data, if necessary.

(*iii*) Use of non-programmable calculator is allowed.

(*iv*) Answer Q. Nos. **1** *or* **2**, Q. Nos. **3** *or* **4**, Q. Nos. **5** *or* **6**, Q. Nos. **7** *or* **8**.

1. (*a*) Solve any *two* of the following : [8]

(*i*) $(D^2 - 4D + 4)\, y = e^{2x} \sin 3x$

(*ii*) $\dfrac{d^2y}{dx^2} + 4y = \tan 2x$ (by variation of parameters)

(*iii*) $(x+1)^2 \dfrac{d^2y}{dx^2} + (x+1)\dfrac{dy}{dx} + y = 2 \sin [\log (x+1)]$.

(*b*) Solve the following system of equations by Gauss-Jordan method : [4]

$$x_1 + x_2 + x_3 = 9$$
$$2x_1 - 3x_2 + 4x_3 = 13$$
$$3x_1 + 4x_2 + 5x_3 = 40.$$

P.T.O.

Or

2. (*a*) Find the elastic curve of a uniform cantilever beam of length l, having a constant weight w kg per foot and determine the deflection of the free end. [4]

(*b*) Using fourth order Runge-Kutta method, solve the equation $\frac{dy}{dx} = \sqrt{x + y}$ subject to the conditions $x = 0$, $y = 1$ and find y at $x = 0.2$ taking $h = 0.2$. [4]

(*c*) Solve the following system of equations by Cholesky's method : [4]

$$4x_1 - 2x_2 = 0$$
$$-2x_1 + 4x_2 - x_3 = 1$$
$$-x_2 + 4x_3 = 0.$$

3. (*a*) Calculate the first four central moments for the following data : [4]

x	f
1	1
2	6
3	13
4	25
5	30
6	22
7	9
8	5
9	2

(*b*) If the probability that an individual suffers a bad reaction from a certain injection is 0.001, determine the probability that out of 2000 individuals, more than 2 individuals will suffer a bad reaction. [4]

(*c*) Find the directional derivative of $\phi = 5x^2y - 5y^2z + 2z^2x$ at the point (1, 1, 1) in the direction of the line : [4]

$$\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}.$$

Or

4. (*a*) Prove the following (any *one*) : [4]

(*i*) $\nabla \cdot \left[r \, \nabla \left(\frac{1}{r^n} \right) \right] = \frac{n(n-2)}{r^{n+1}}$

(*ii*) For scalar functions ϕ and ψ show that :

$$\nabla \cdot [\phi \nabla \psi - \psi \nabla \phi] = \phi \nabla^2 \psi - \psi \nabla^2 \phi.$$

(*b*) Show that $\bar{\text{F}} = (ye^{xy} \cos z)\bar{i} + (xe^{xy} \cos z)\bar{j} - (e^{xy} \sin z)\bar{k}$ is irrotational. Find ϕ such that $\bar{\text{F}} = \nabla \phi$. [4]

(*c*) If $\bar{x} = 8.2, \bar{y} = 12.4, \sigma_x = 6.2, \sigma_y = 20, r(x, y) = 0.9$. Find lines of regression. Also estimate the value of x for $y = 10$ and value of y for $x = 10$. [4]

5. Solve any *two* :

(*a*) Using Green's theorem to evaluate $\int_{\text{C}} (3y \, dx + 2x \, dy)$, where C is boundary $0 \le x \le \pi, \; 0 \le y \le \sin x$. [7]

(*b*) Using Divergence theorem to evaluate

$$\iint_S [(2x + 3z)i - (xz + y)\, j + (y^2 - 2z)k] \,.\, \bar{d}S$$

where S is the surface of sphere having center at (3, –1, 2) and radius is 3. [6]

(*c*) Evaluate $\iint_S (\nabla \times \bar{F}) \,.\, \bar{d}S$ for the surface of paraboloid $z = 4 - x^2 - y^2,\ z \geq 0$ and $\bar{F} = y^2 i + zj + xyk.$ [6]

Or

6. Solve any *two* :

(*a*) Find the workdone in moving a particle along the curve $\bar{r} = a \cos \theta i + a \sin \theta j + b\theta k$ from $\theta = \frac{\pi}{4}$ to $\theta = \frac{\pi}{2}$ under the force field is given by : [7]

$$\bar{F} = (-3a \sin^2 \theta \,.\cos \theta)\, i + a(2 \sin \theta - 3 \sin^3 \theta) j + b \,.\, \sin 2\theta k.$$

(*b*) Show that : [6]

$$\int_C [\bar{u} \times (\bar{r} \times \bar{v})] \,.\bar{d}r = -(\bar{u} \times \bar{v}) \,.\, \iint_S dS$$

Where S is the open surface bounded by curve 'C' and $\bar{u}, \bar{v}$ are constant vectors.

(*c*) Using Divergence theorem to show that : [6]

$$\iint_S \bar{r} \,.\, \bar{n}\, dS = 3V$$

where V is the volume enclosed by 'S'.

7. Solve any *two* of the following :

(*a*) A tightly stretched string with fixed ends $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $3x(l - x)$ for $0 < x < l$, find the displacement. [7]

(*b*) Solve : [6]

$$\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$$

if :

(*i*) u is finite for all t

(*ii*) $u(0, t) = 0$ for all t

(*iii*) $u(l, t) = 0$ for all t

(*iv*) $u(x, 0) = u_0$ for $0 \leq x \leq l$.

(*c*) Solve the equation $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$ with conditions : [6]

(*i*) $V = 0$ when $y \to \infty$ for all x

(*ii*) $V(0, y) = 0$ for all y

(*iii*) $V(1, y) = 0$ for all y

(*iv*) $V(x, 0) = x(1 - x)$ for $0 < x < 1$.

Or

8. Solve any *two* of the following :

(*a*) A taut string of a length $2l$ is fastened at both ends. The midpoint of the string is taken to a height b and then released from rest in that position, obtain the displacement. [7]

(*b*) Solve $\frac{\partial V}{\partial t} = k\frac{\partial^2 V}{\partial x^2}$ if : [6]

(*i*) V is finite as $t \to \infty$

(*ii*) $\left(\frac{\partial V}{\partial x}\right)_{x=0} = 0$ for all t

(*iii*) $V(l, t) = 0$ for all t

(*iv*) $V(x, 0) = V_0$ for $0 < x < l$.

(*c*) A thin sheet of metal bounded by the x-axis and the lines $x = 0$, $x = 1$ and stretching to infinity in the y-direction has its upper and lower faces perfectly insulated and its vertical edges and edge at infinity are maintained at the constant temperature 0°C, while over the base temperature of 100°C is maintained. Find steady-state temperature $u(x, y)$. [6]